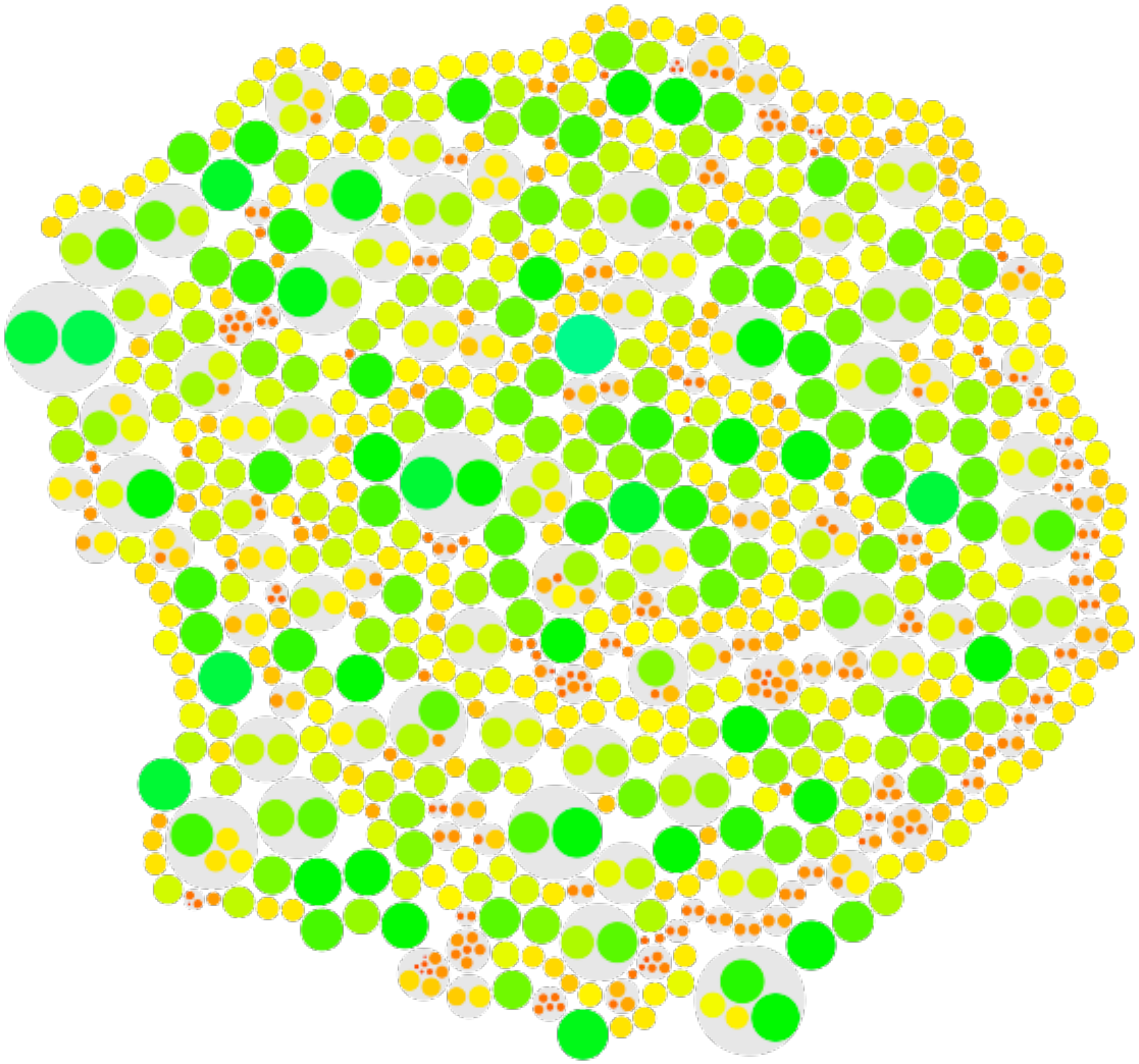
Institute for Advanced Study

The case for stochastic orbital migration

Hanno Rein @ Kobe, August 2013

Extra-solar planet census

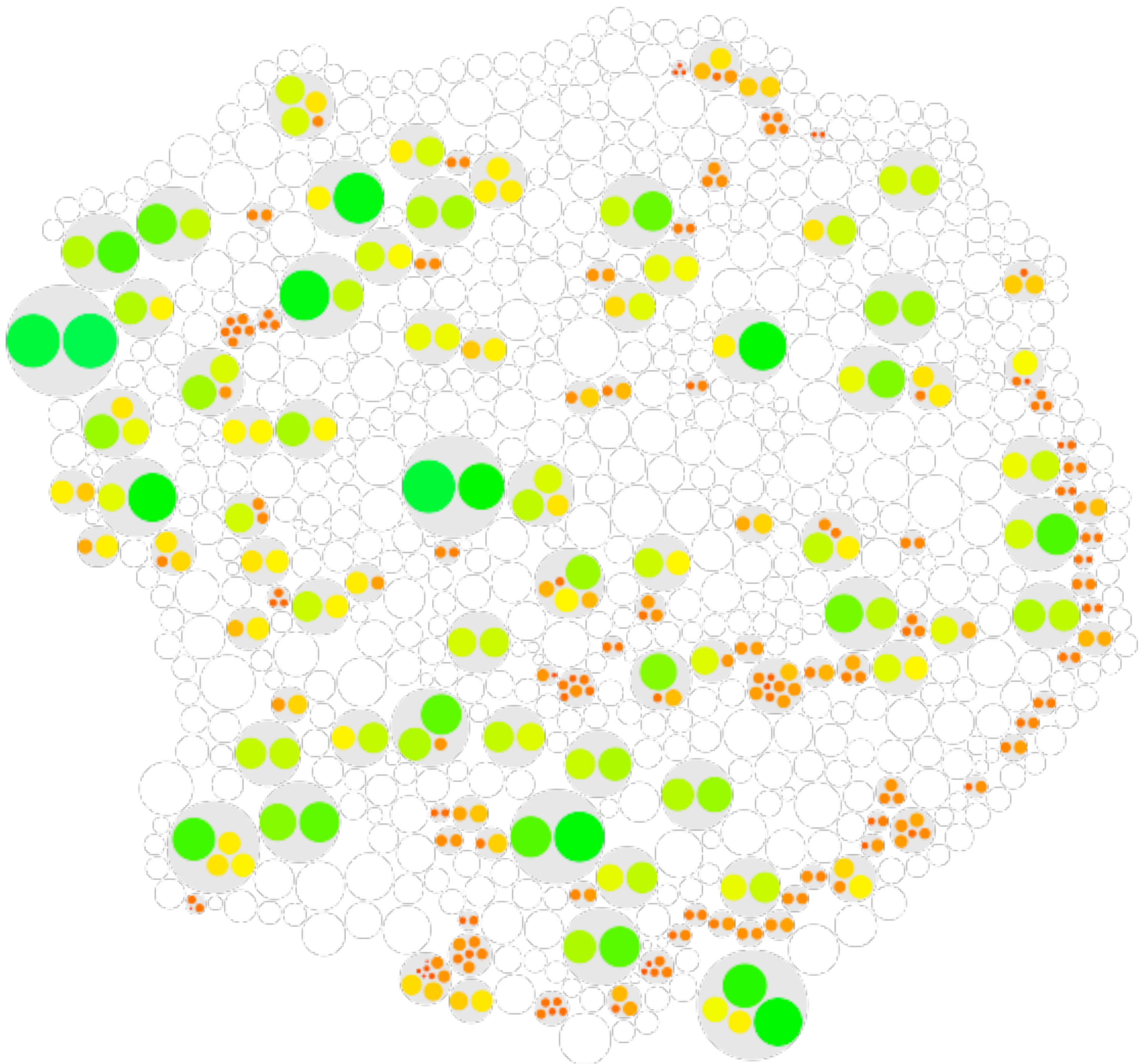
All discovered extra-solar planets



869 confirmed extra-solar planets

- Super-Jupiters
- (Hot) Jupiters
- Neptunes
- Super-Earths
- Earth-like planets

All multi-planetary systems

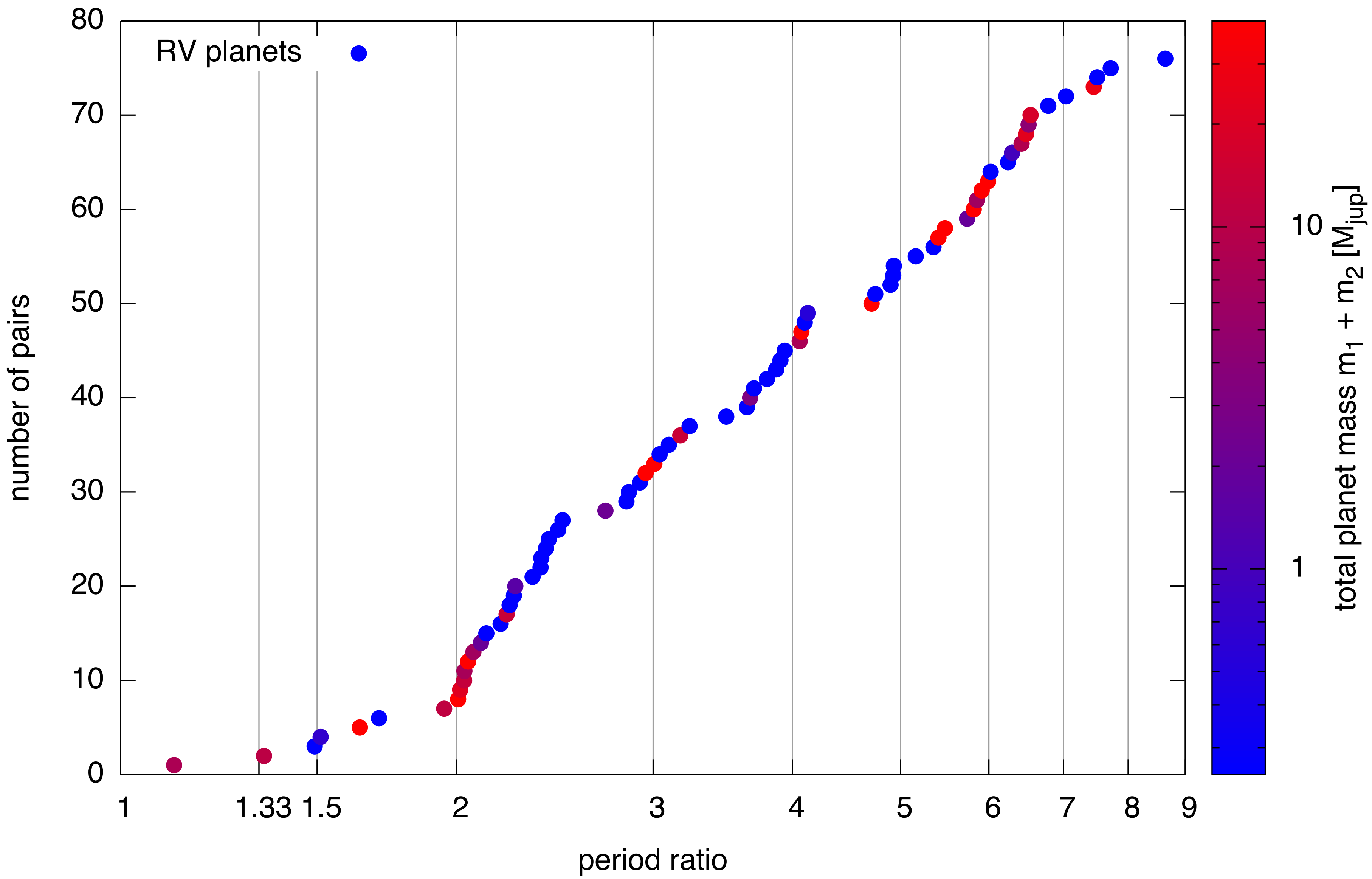


327 confirmed planets in multi-planetary systems

- Multiple Jupiters
- Densely packed systems of Neptunes and (Super)-Earths
- 1 Solar System
- Some systems are deep in resonance

Open Exoplanet Catalogue (Rein 2012b)

Radial velocity planets

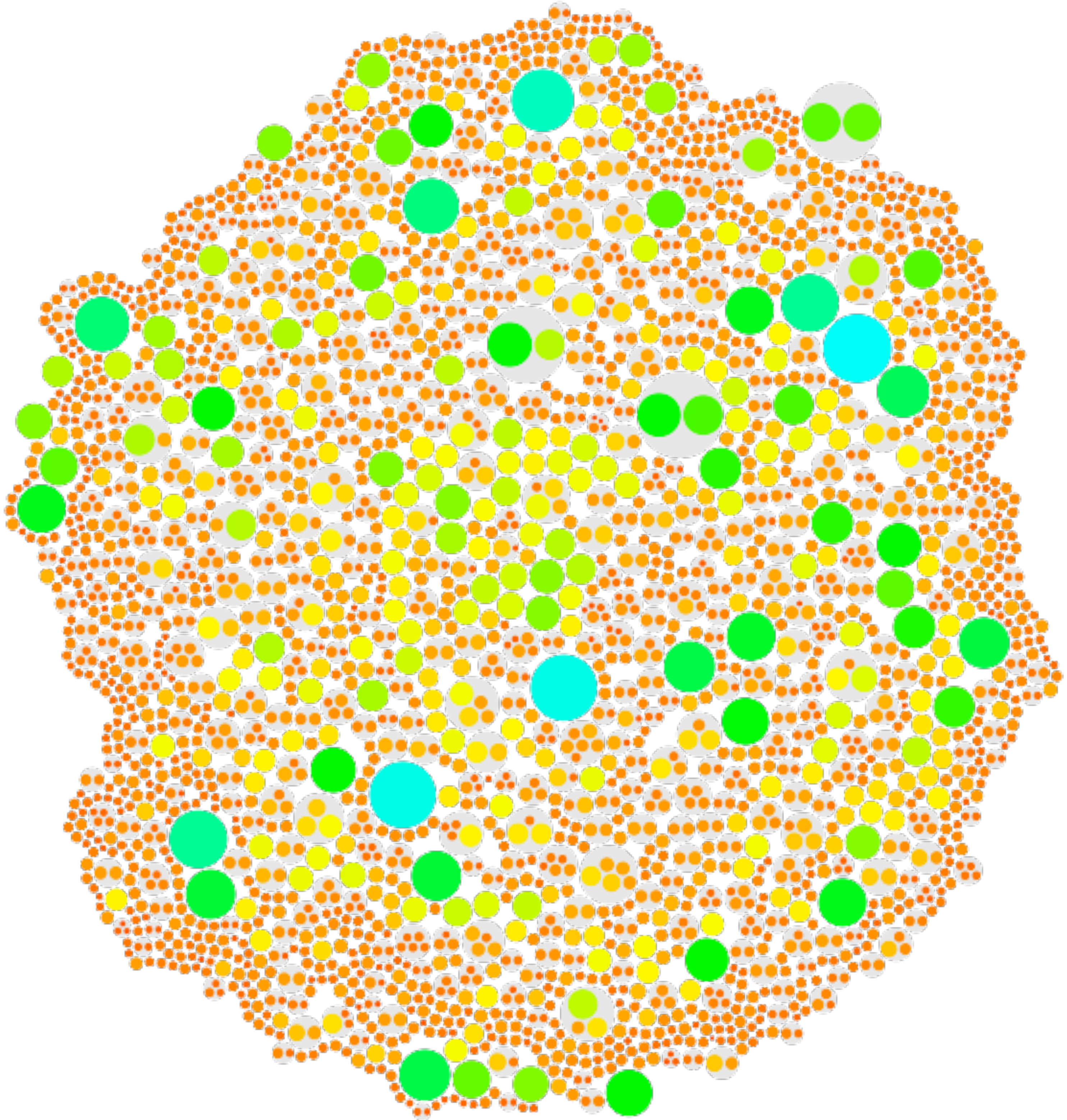


Cumulative period ratio in multi-planetary systems

- Periods of systems with massive planets tend to pile up near integer ratios
- Most prominent features at 4:1, 3:1, 2:1, 3:2

Rein, Payne, Veras & Ford (2012)

Kepler candidates

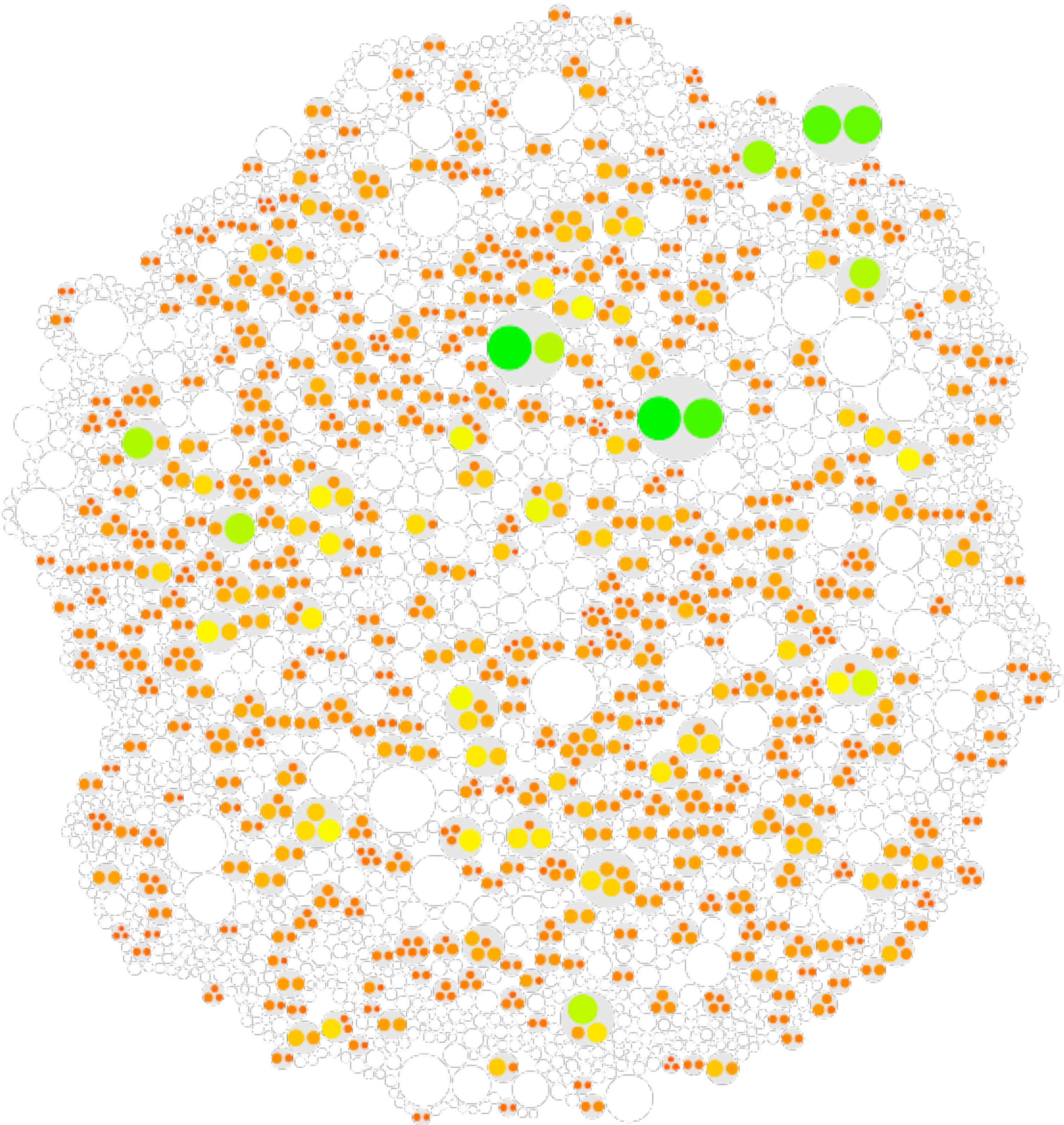


2740 planet candidates

- Probing a different regime
- Small mass planets
- A lot of planets

Open Exoplanet Catalogue (Rein 2012b)

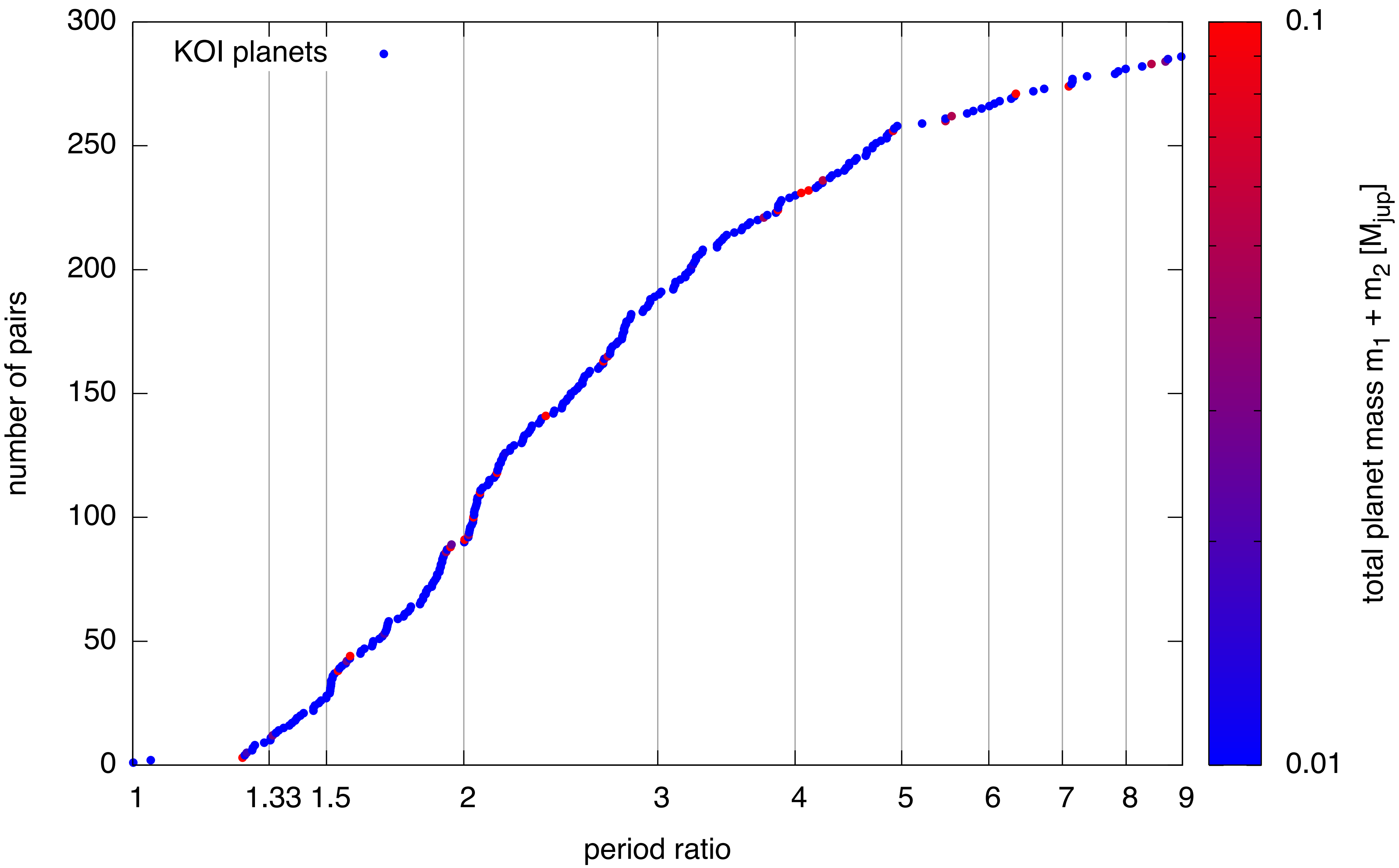
Kepler candidates with multiple planets



Kepler multi-planetary systems

- Small mass planets
- Hierarchical systems
- Densely packed
- Not many are in resonance

Kepler's transiting planet candidates

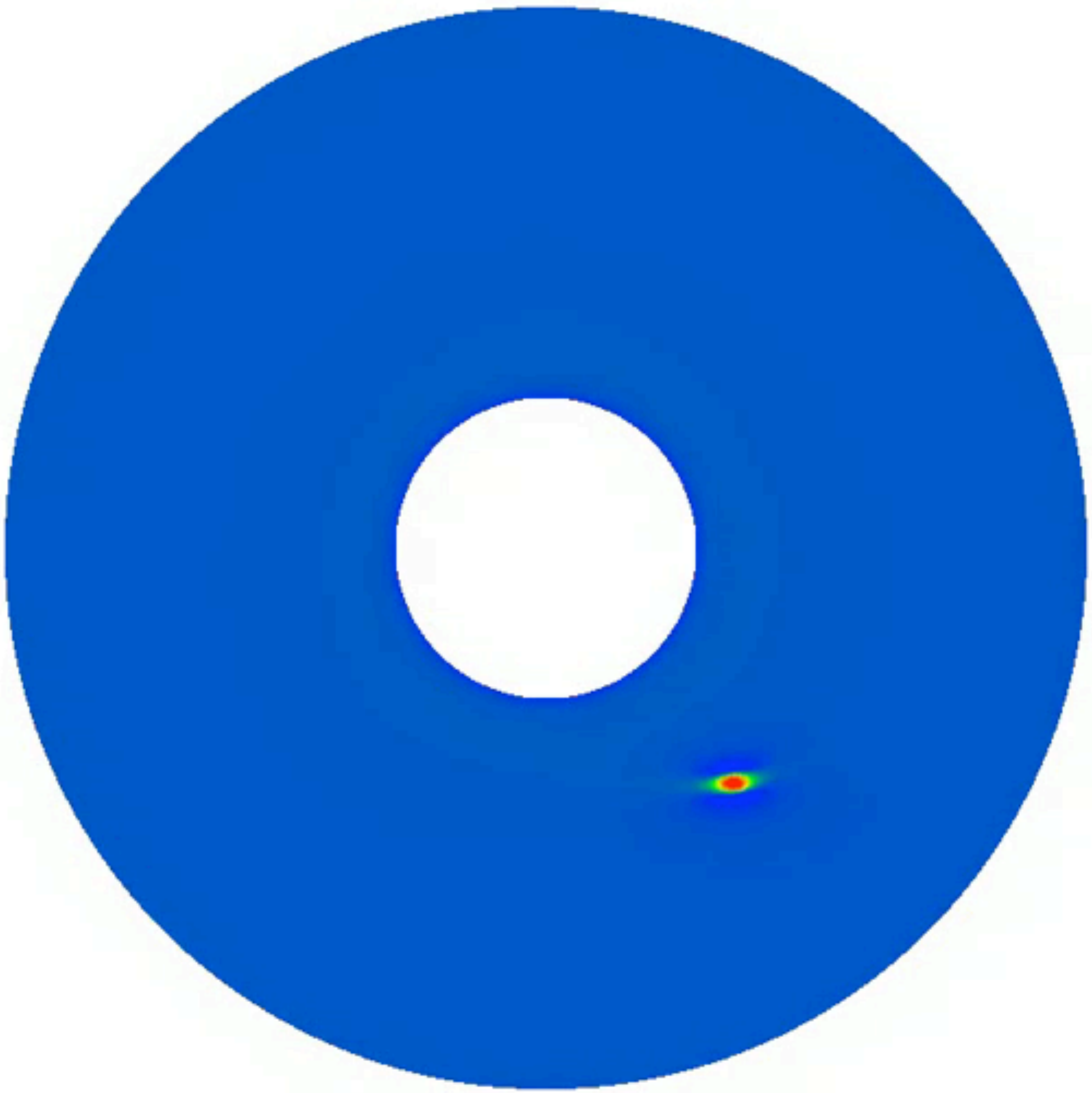


- Period ratio distribution much smoother for small mass planets
- Deficiencies near 4:3, 3:2, 2:1
- Excess slightly outside of the exact commensurability

Stochastic orbital migration

Migration - Type I

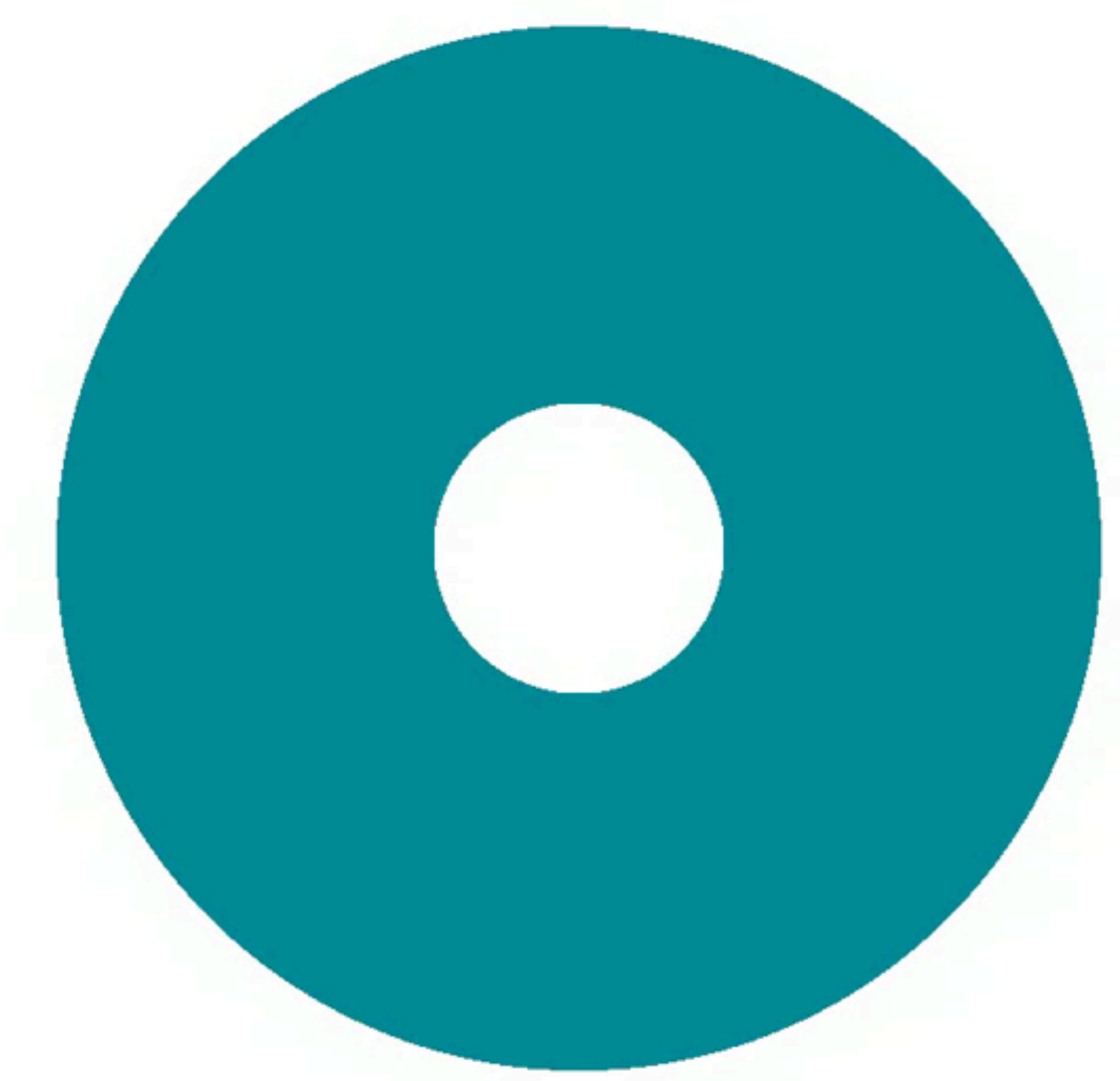
- Low mass planets
- No gap opening in disc
- Migration rate is fast
- Depends strongly on thermodynamics of the disc



2D hydro code Prometheus (Rein 2010)

Migration - Type II

- Massive planets (typically bigger than Saturn)
- Opens a (clear) gap
- Migration rate is slow
- Follows viscous evolution of the disc



2D hydro code Prometheus (Rein 2010)

How does a real protoplanetary disk look like?

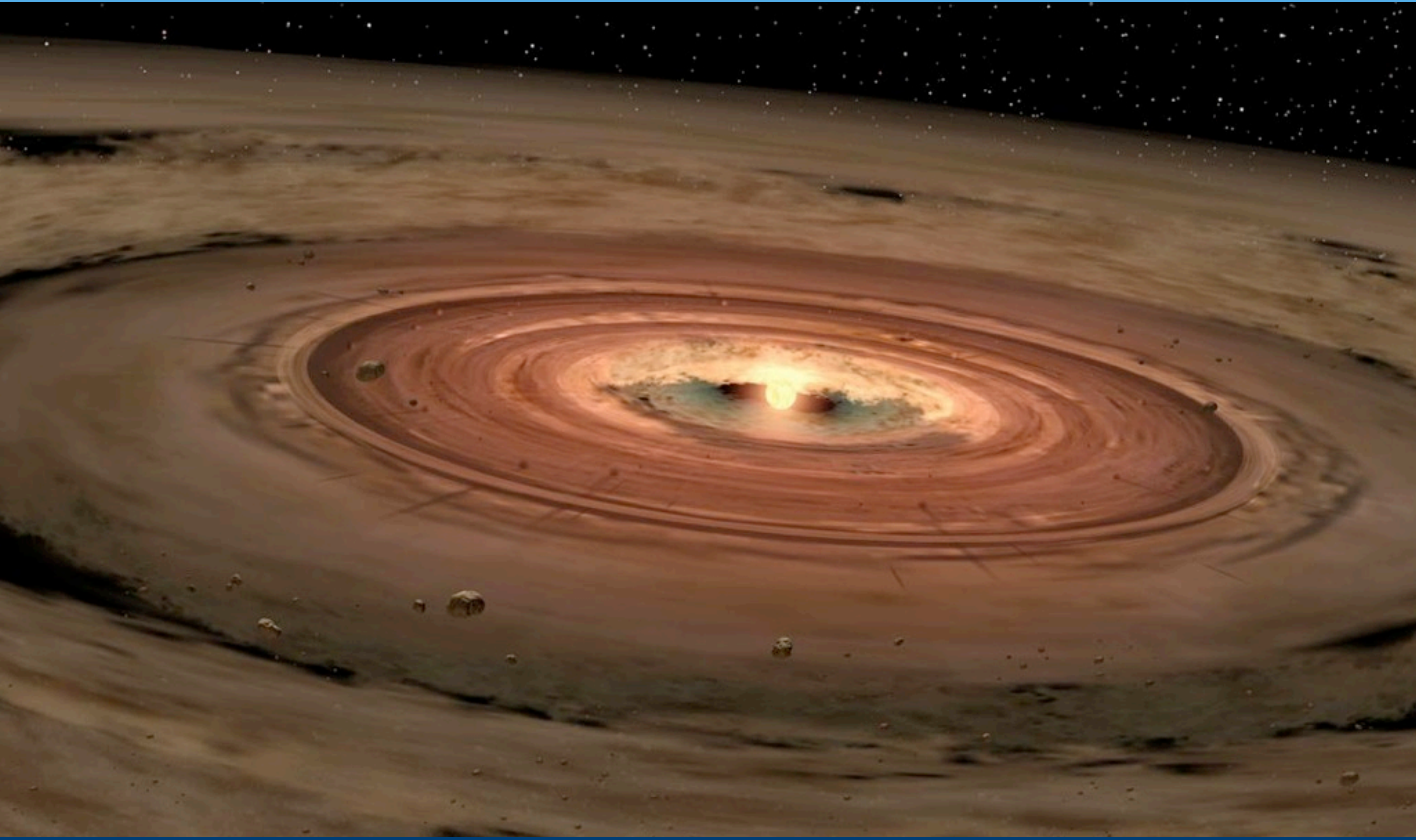
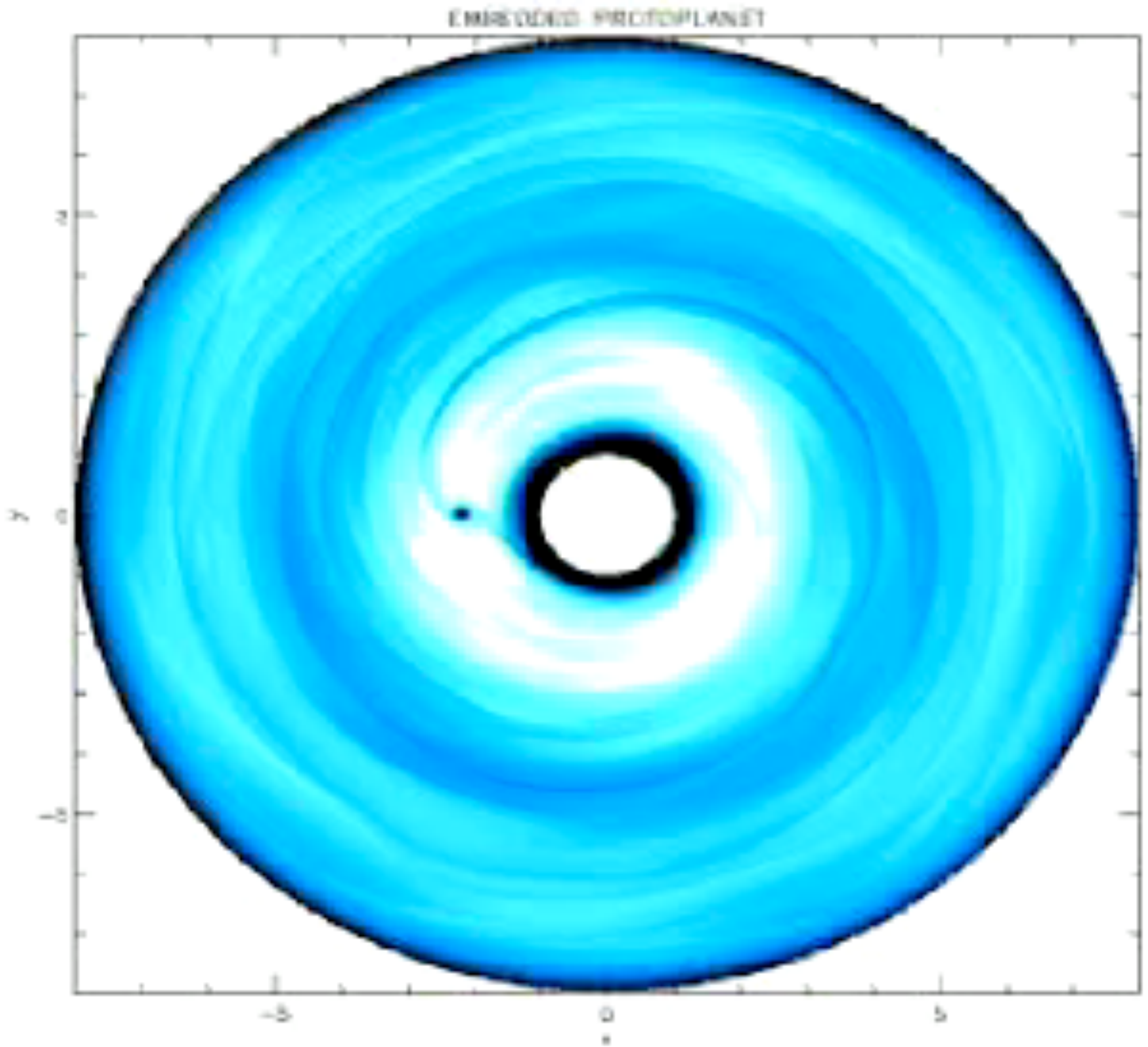


Image credit: NASA/JPL-Caltech

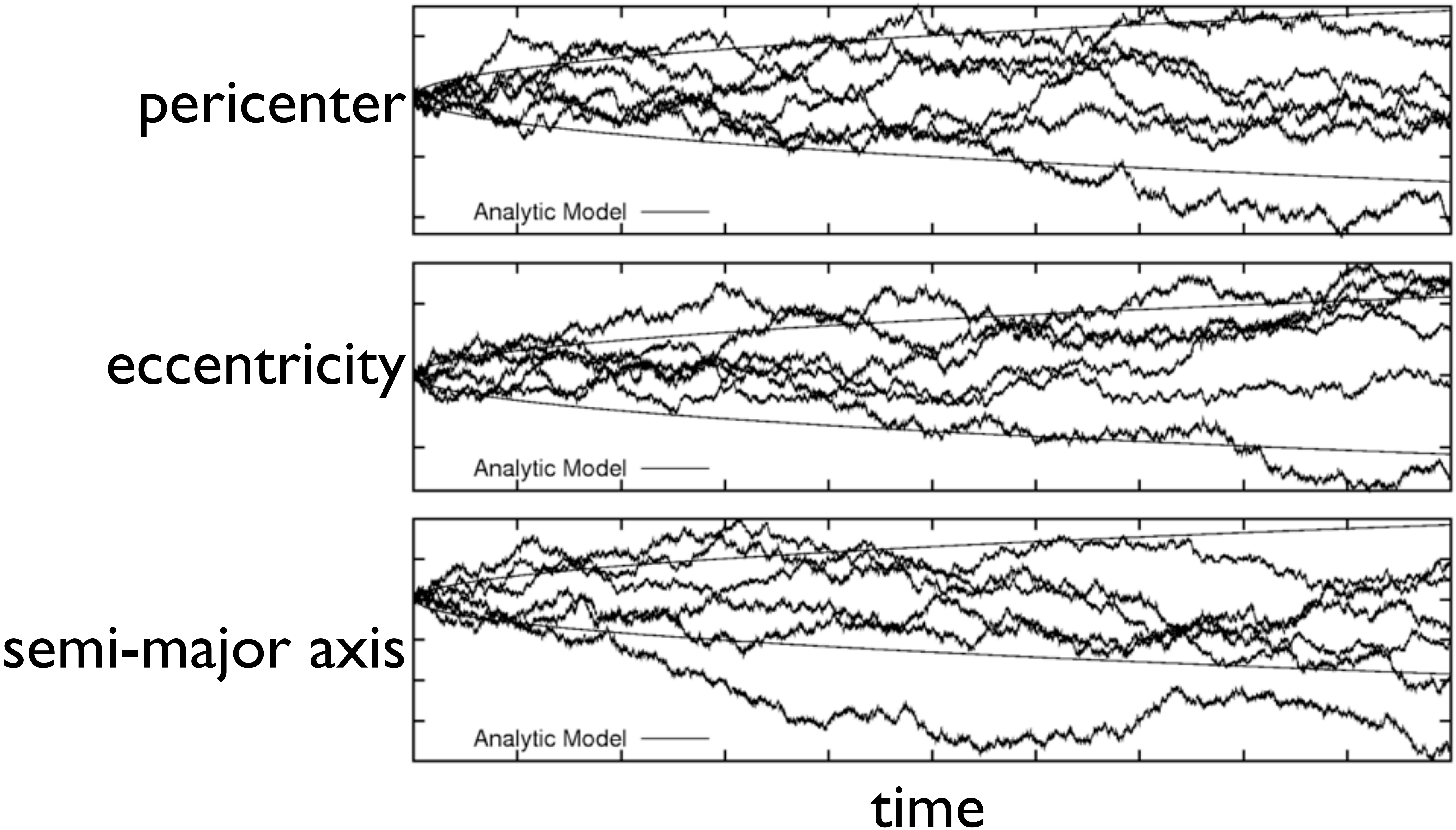
Why think about stochastic migration?

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties in strength of forces



Animation from Nelson & Papaloizou 2004
Random forces measured by Laughlin et al. 2004, Nelson 2005, Oischi et al. 2007

Random walk in all orbital parameters

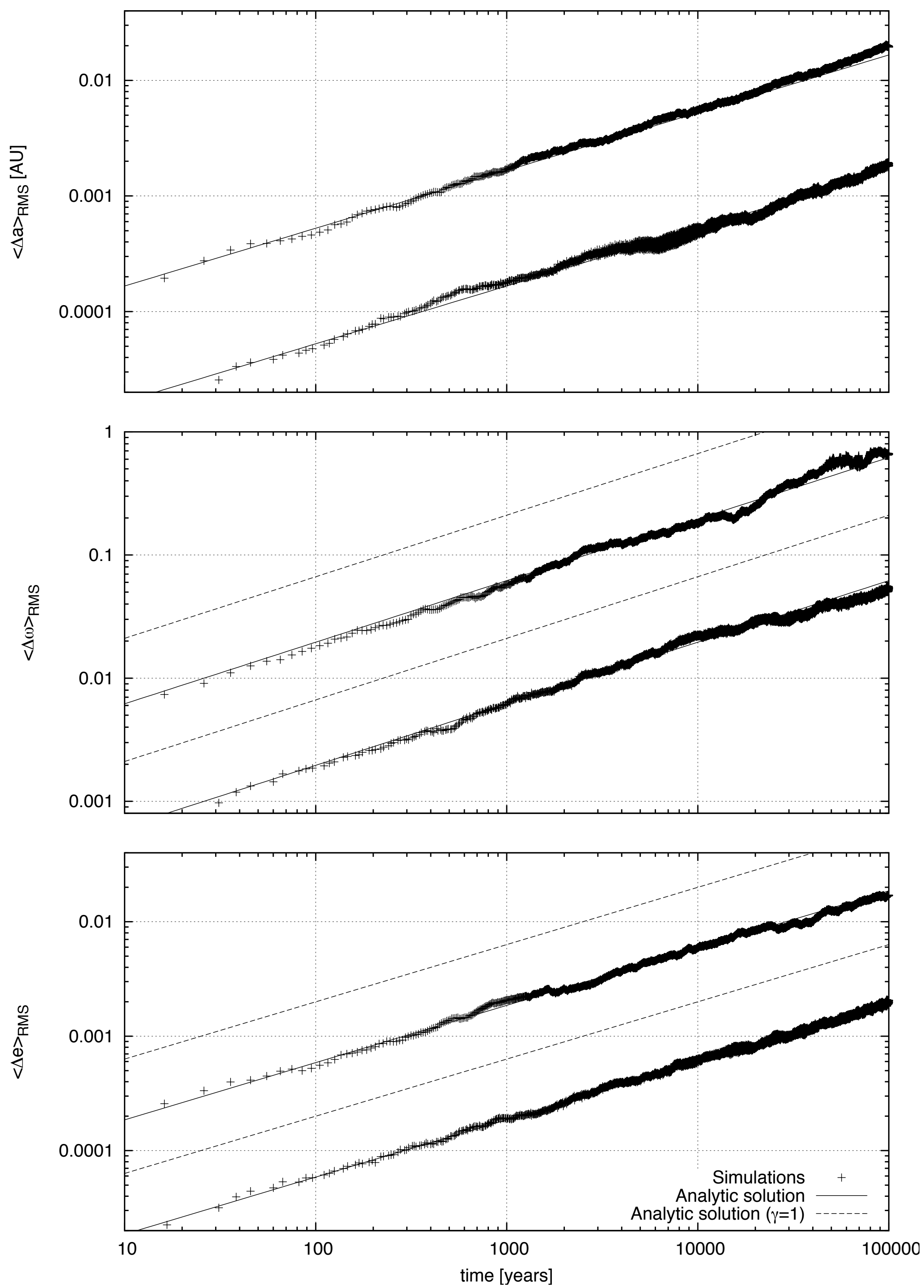


Analytic growth rates for 1 planet

$$(\Delta a)^2 = 4\frac{Dt}{n^2}$$

$$(\Delta \varpi)^2 = \frac{2.5}{e^2}\frac{\gamma Dt}{n^2a^2}$$

$$(\Delta e)^2 = 2.5\frac{\gamma Dt}{n^2a^2}$$

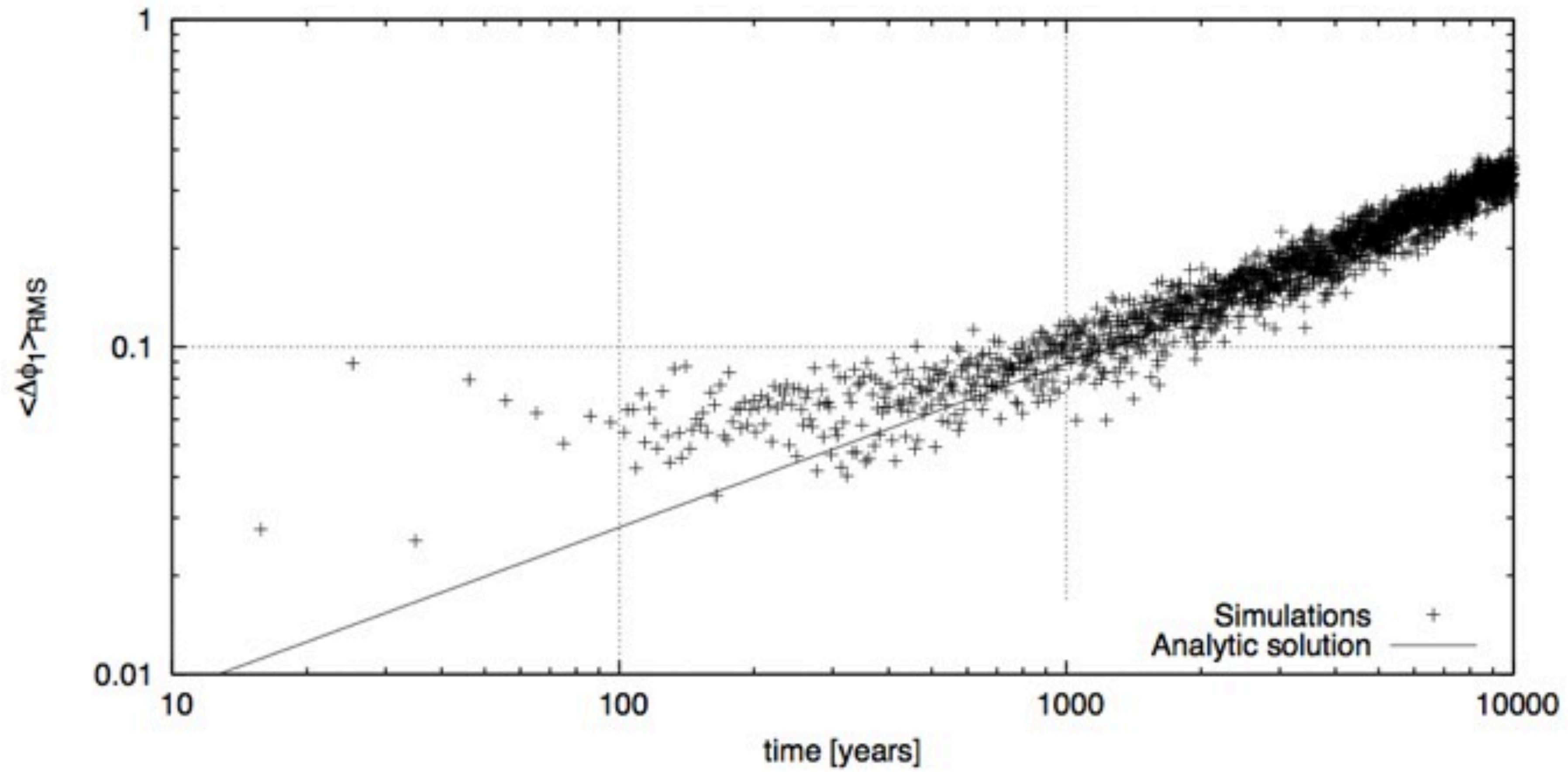


Rein & Papaloizou 2009, Adams et al 2009, Rein 2010

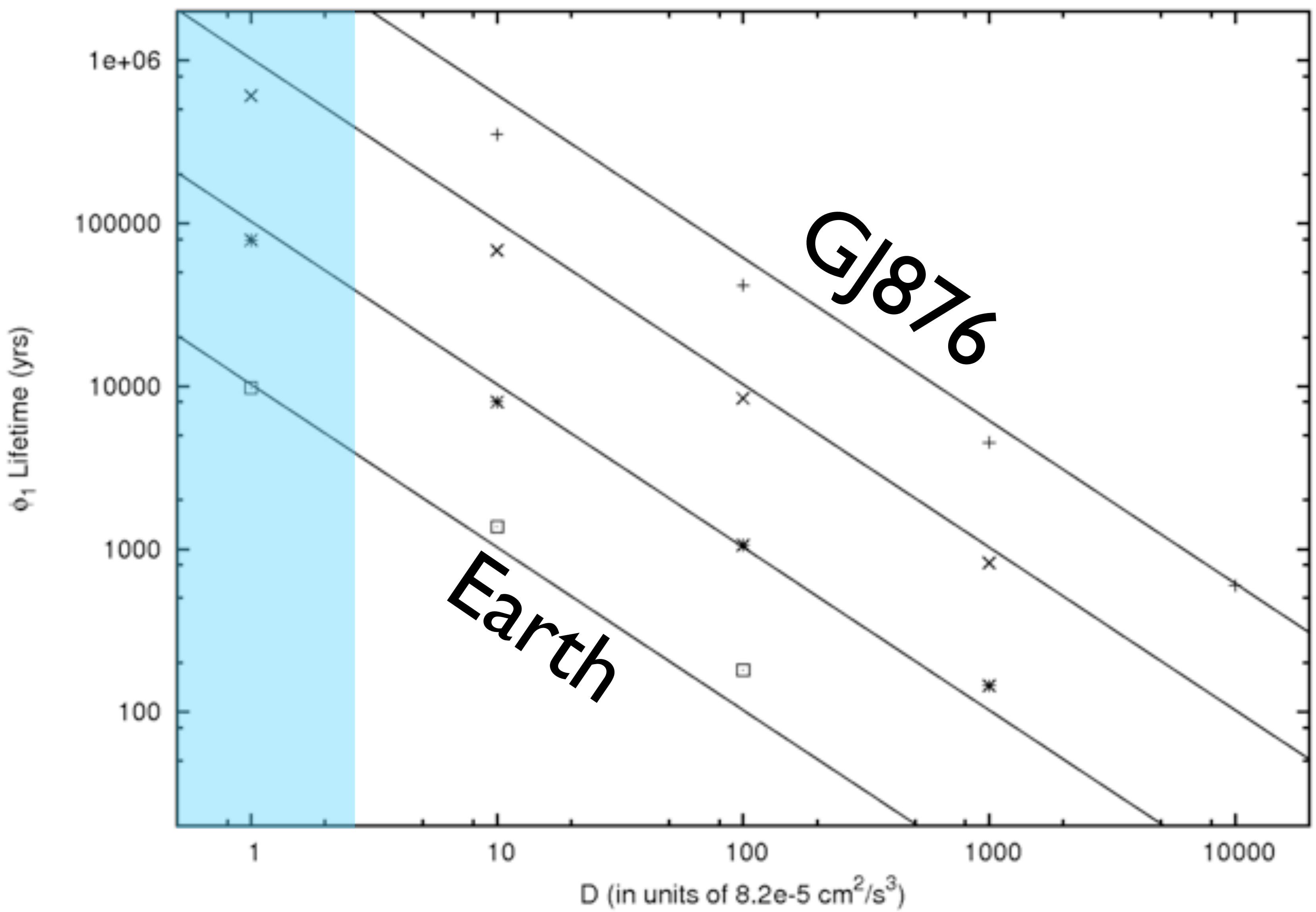
Analytic growth rates for 2 planets

$$\frac{(\Delta\phi_1)^2}{(p+1)^2} = \frac{9\gamma_f}{a_1^2\omega_{lf}^2}\,D\,t$$

$$(\Delta(\Delta\varpi))^2 = \frac{5\gamma_s}{4a_1^2 n_1^2 e_1^2}\,D\,t$$



Multi-planetary systems in mean motion resonance

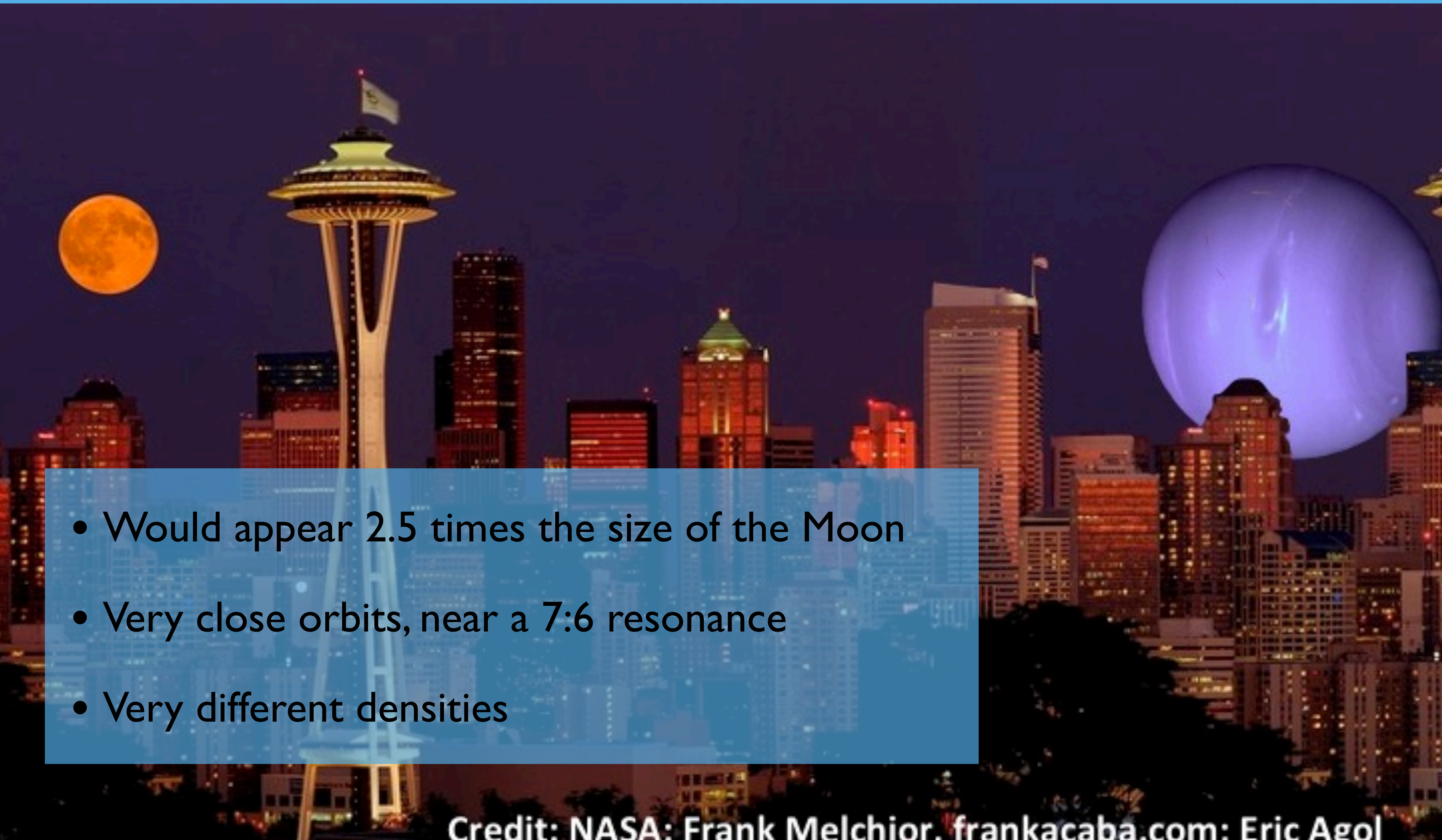


- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable for entire disc lifetime

Rein & Papaloizou 2009

The formation of Kepler-36

Kepler-36 c as seen from Kepler-36 b

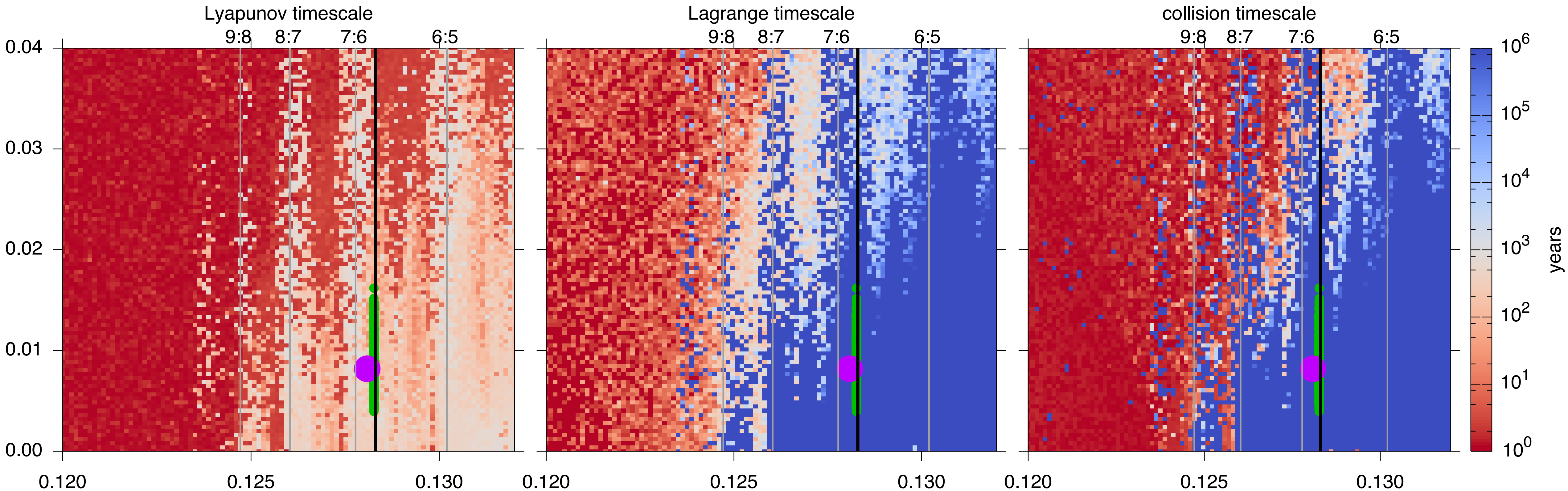


- Would appear 2.5 times the size of the Moon
- Very close orbits, near a 7:6 resonance
- Very different densities

Stability of Kepler-36

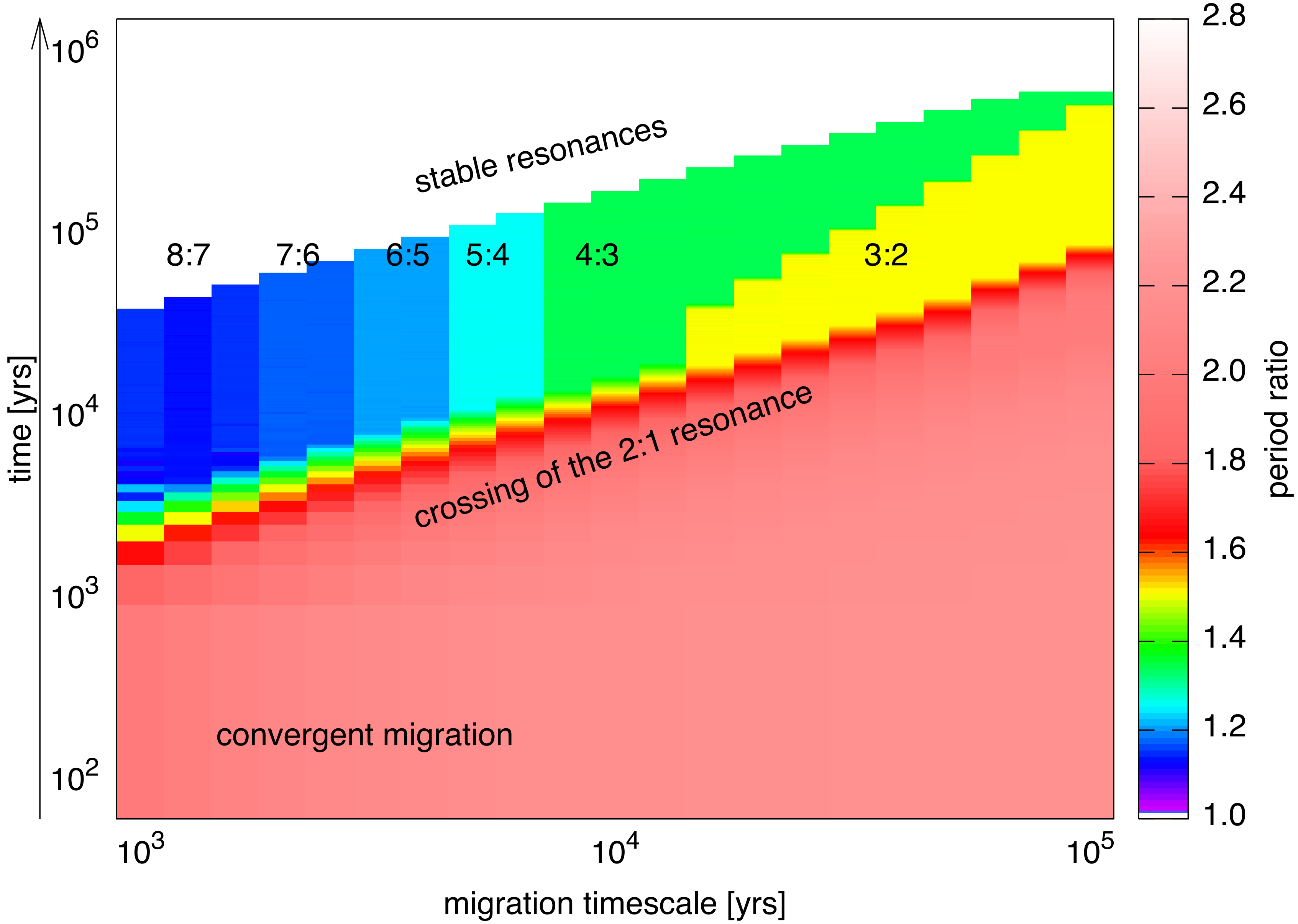
Lyapunov timescale | Lagrange timescale | Collision timescale

eccentricity



semi-major axis | semi-major axis | semi-major axis

Formation of Kepler-36



- Migration rate and mass ratio determine the final resonance
- Higher order resonances require faster migration rates
- Higher mass planets end up in lower order resonances
- Once in resonance, planets often stay there for the rest of the disc lifetime

Problem with Kepler-36

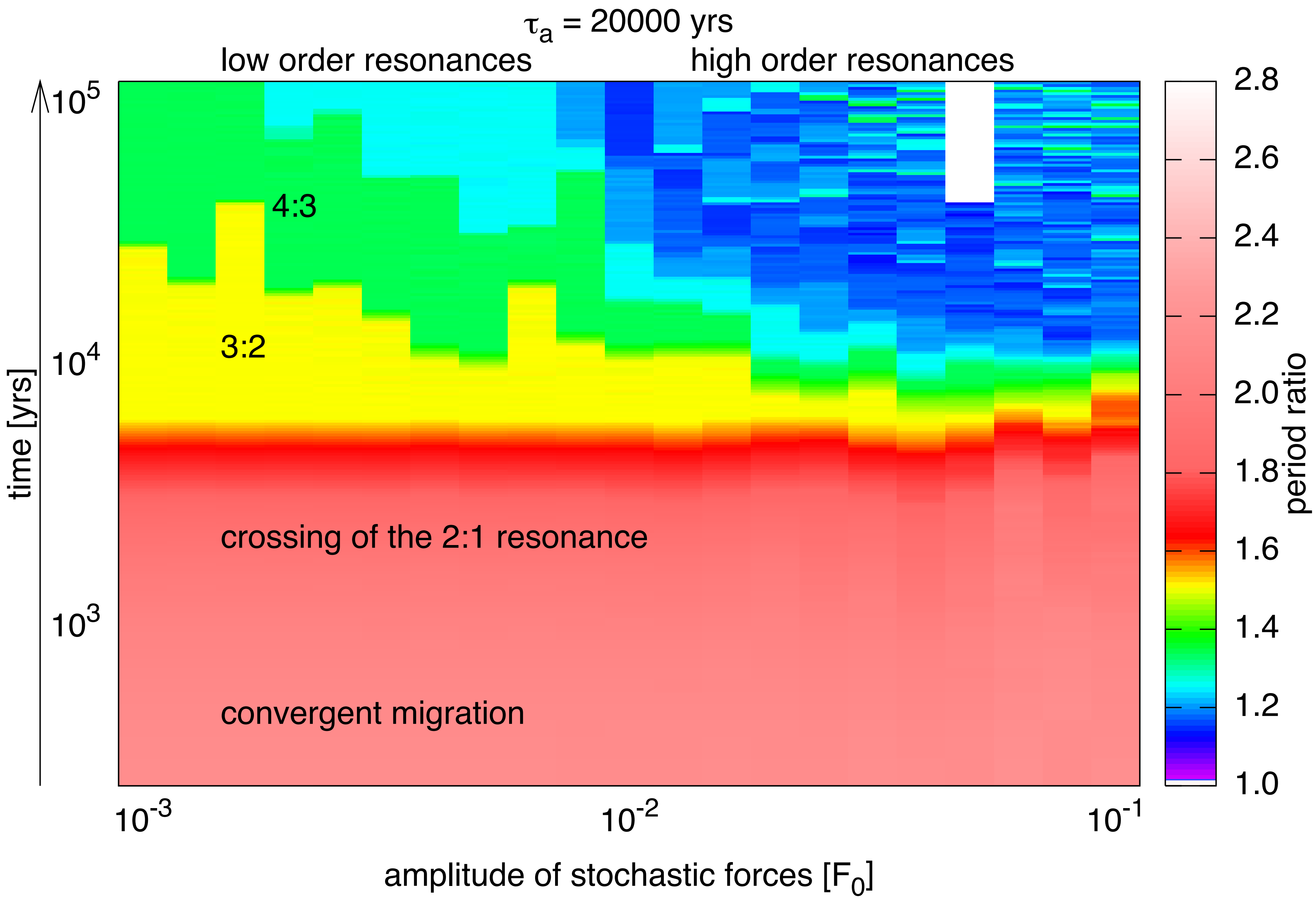
<1000 years

Need extremely fast migration rate to capture into a high order resonance.

Unrealistically fast.

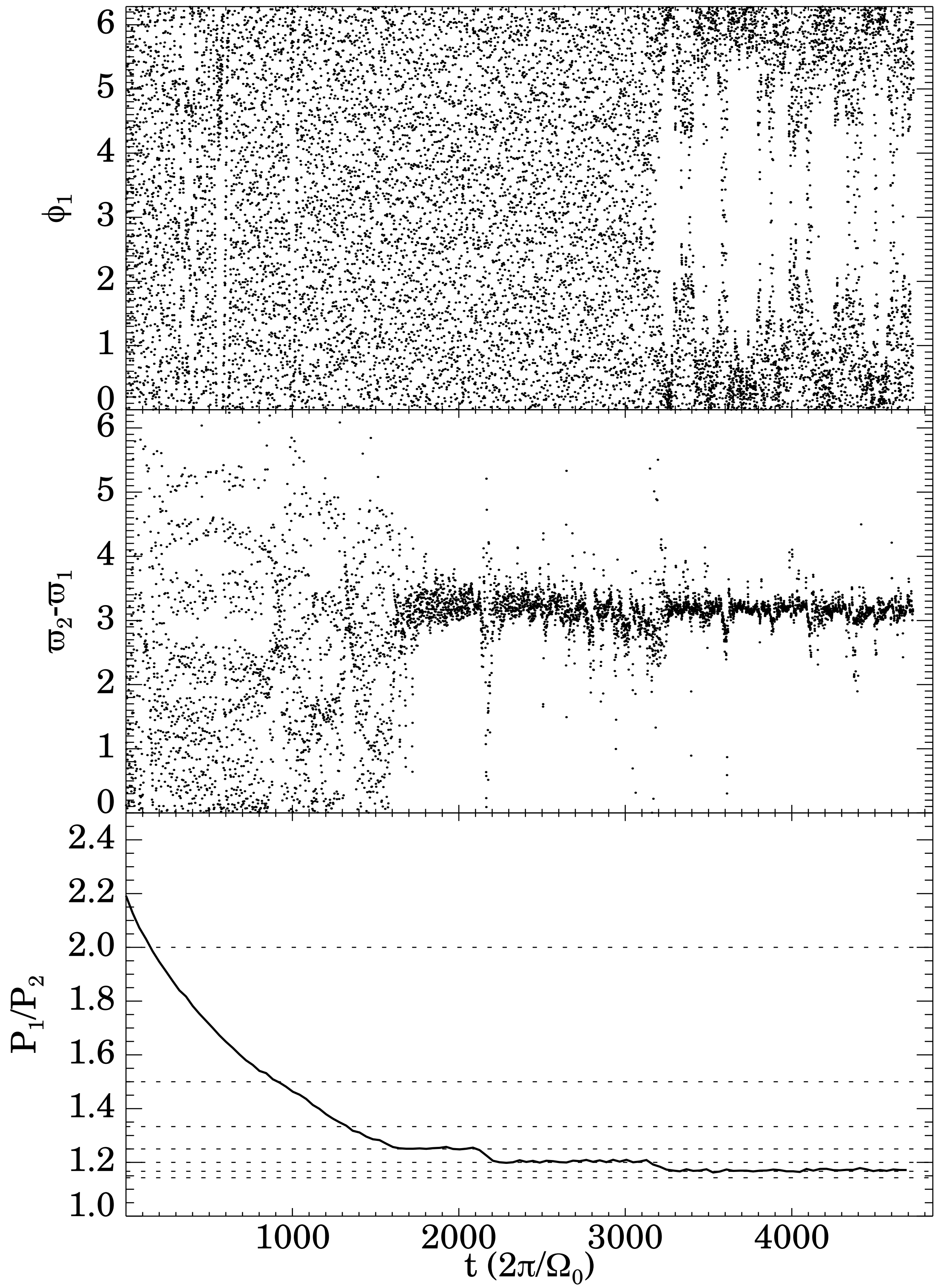
Planets are not large enough to migrate in Type III regime.

Solution: Stochastic migration



Pardekooper, Rein & Kley (in press)

Hydrodynamic simulation



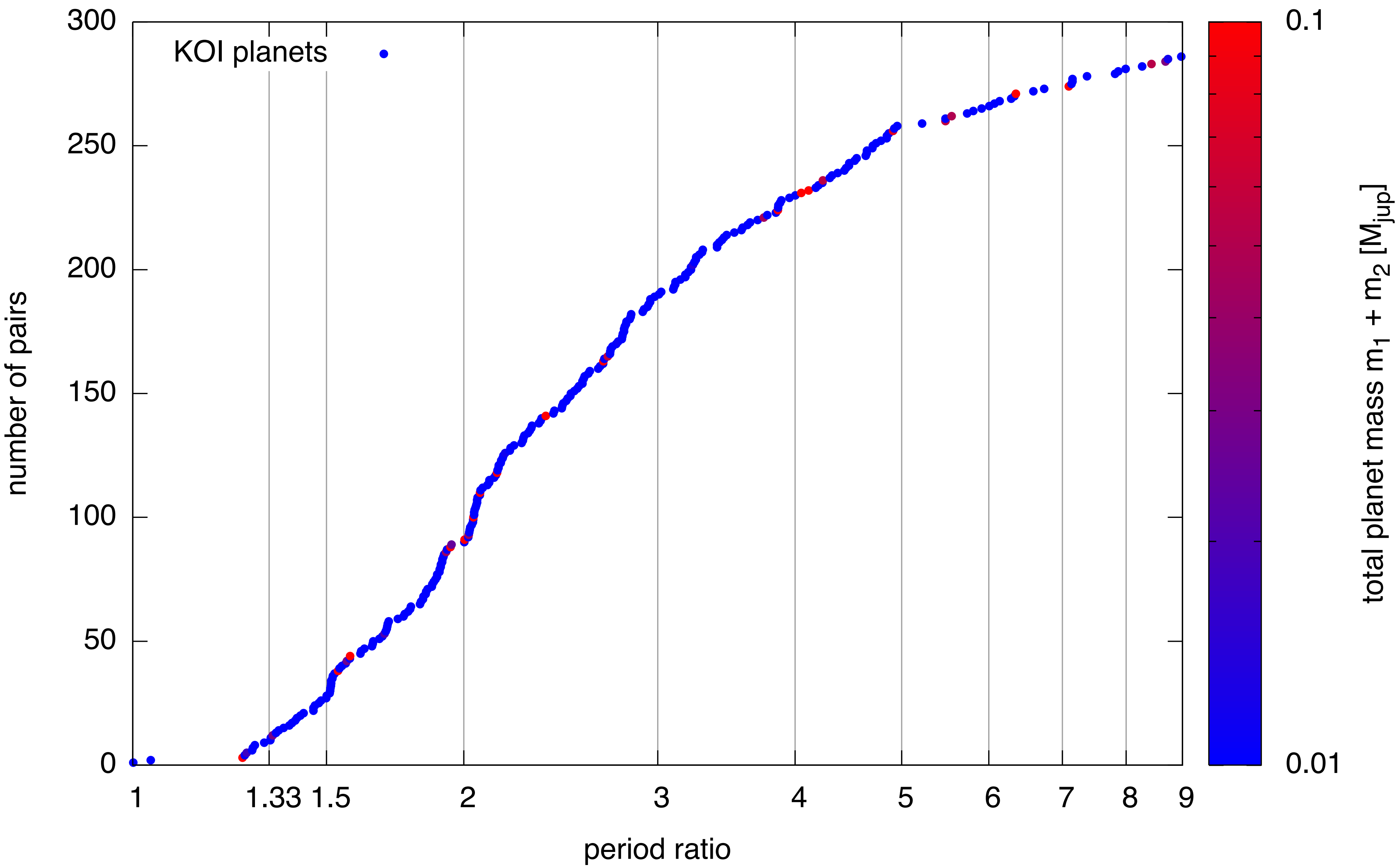
Resonant angle

Resonant angle

Period ratio

A statistical analysis

Kepler's transiting planet candidates



- Period ratio distribution much smoother for small mass planets
- Deficiencies near 4:3, 3:2, 2:1
- Excess slightly outside of the exact commensurability

Rein, Payne, Veras & Ford (2012)

Testing stochastic migration: Method

Architecture and masses from observed KOIs

Placing planets in a MMSN, further out, further apart, randomizing all angles

N-body simulation with migration forces

Testing stochastic migration: Advantages

Comparison of statistical quantities

- Period ratio distribution
- Eccentricity distribution
- TTVs

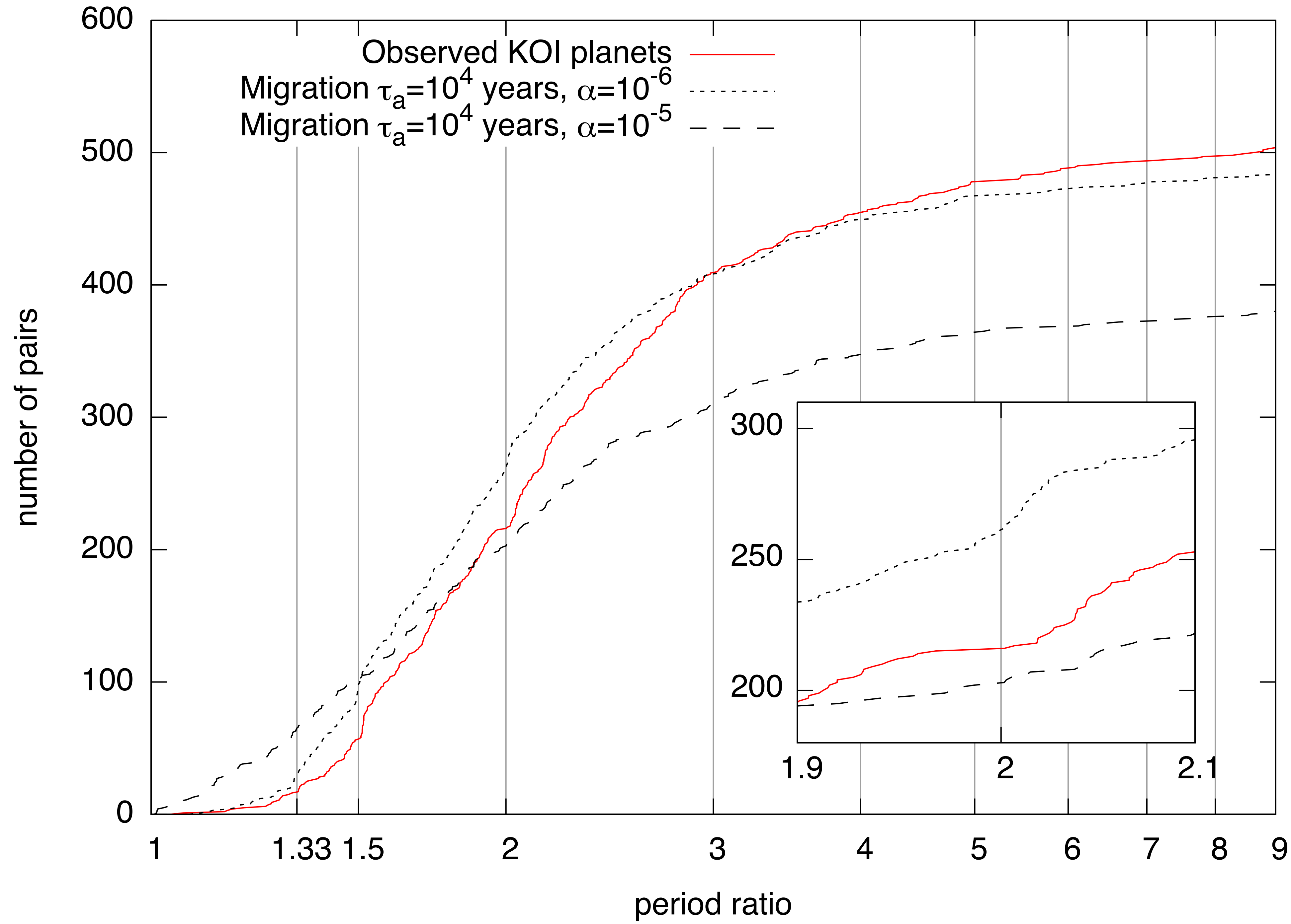
Comparison of individual systems

- Especially interesting for multi-planetary systems
- Can create multiple realizations of each system

No synthesis of a planet population required

- Observed masses, architectures
- Model independent

Preliminary results



Future expansions

Physical disk model

- 1D hydrodynamic simulation
- Coupled to N-body simulations

Completeness

- Include planets missed by Kepler

Other physical effects

- Tidal damping
- Evaporation

GPU based integrators

- Allows for much bigger samples
- Wider parameter space exploration

Summary

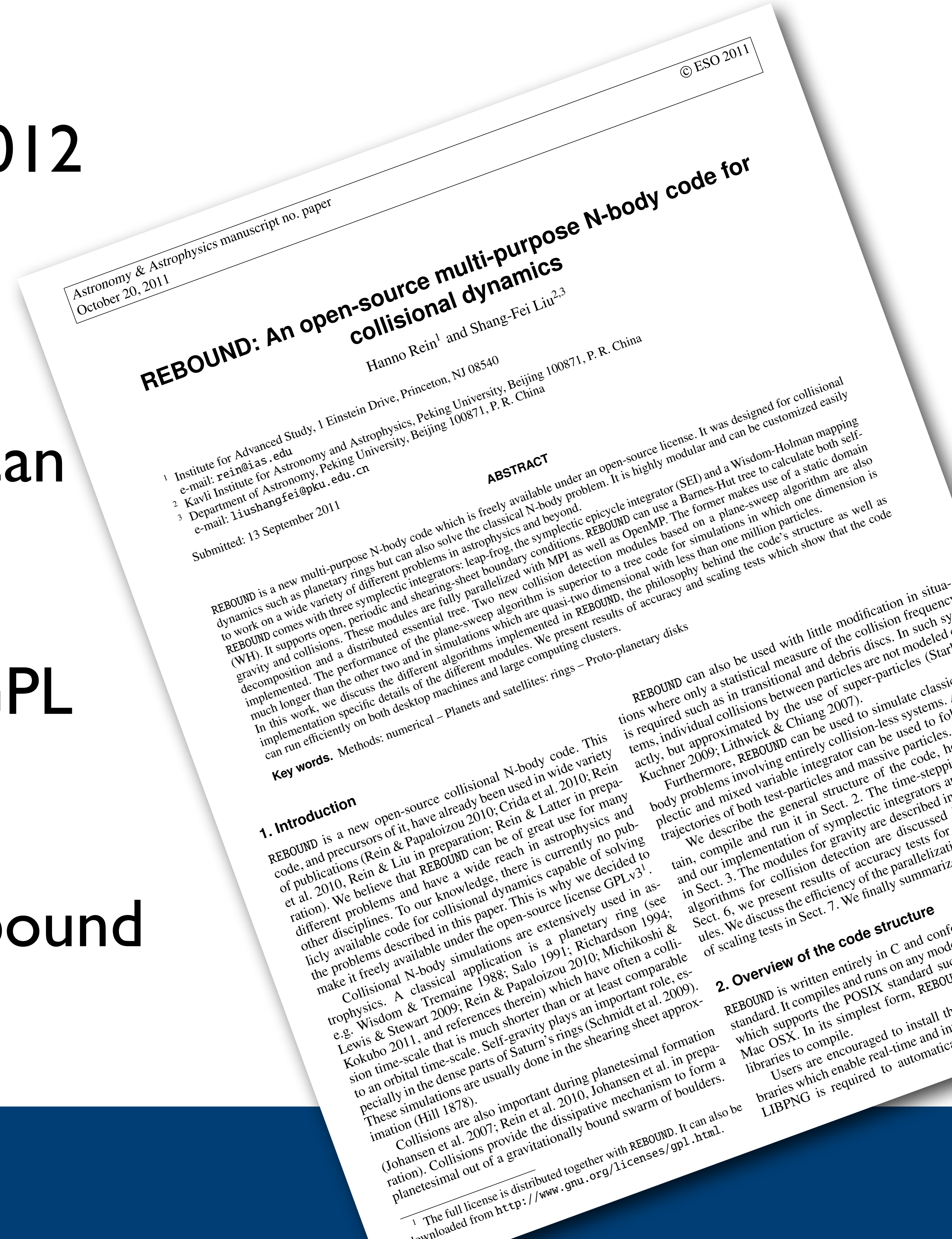
The case for stochastic orbital migration

- Stochastic migration is directly observable in Saturn's rings.
- Protoplanetary disks are turbulent due to the MRI.
- Stochastic migration plays an important role for small mass planets.
- Resonances can easily get destroyed.
- Tendency to form high order resonance.
- Very soon, we will understand how most planets in the Kepler sample formed.

Saturn’s Rings

REBOUND

- Code description paper published by A&A, Rein & Liu 2012
- Multi-purpose N-body code
- Only public N-body code that can be used for granular dynamics
- Written in C99, open source, GPL
- Freely available at http://github.com/hannorein/rebound


Astronomy & Astrophysics manuscript no. paper
October 20, 2011

© ESO 2011

REBOUND: An open-source multi-purpose N-body code for collisional dynamics

Hanno Rein[1] and Shang-Fei Liu[2,3]

[1] Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540
e-mail: rein@ias.edu
[2] Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, P. R. China
[3] Department of Astronomy, Peking University, Beijing 100871, P. R. China
e-mail: liushangfei@pku.edu.cn

Submitted: 13 September 2011

ABSTRACT

REBOUND is a new multi-purpose N-body code which is freely available under an open-source license. It was designed for collisional dynamics such as planetary rings but can also solve the classical N-body problem. It is highly modular and can be customized easily to work on a wide variety of different problems in astrophysics and beyond. REBOUND comes with three symplectic integrators: leap-frog, the symplectic epicycle integrator (SEI) and a Wisdom-Holman mapping (WH). It supports open, periodic and shearing-sheet boundary conditions. REBOUND can use a Barnes-Hut tree to calculate both self-gravity and collisions. These modules are fully parallelized with MPI as well as OpenMP. The former makes use of a static domain decomposition and a distributed essential tree. Two new collision detection modules based on a plane-sweep algorithm are also implemented. The performance of the plane-sweep algorithm is superior to a tree code for simulations in which one dimension is much longer than the other two and in simulations which are quasi-two dimensional with less than one million particles. In this work, we discuss the different algorithms implemented in REBOUND, the philosophy behind the code's structure as well as implementation specific details of the different modules. We present results of accuracy and scaling tests which show that the code can run efficiently on both desktop machines and large computing clusters.

Key words. Methods: numerical – Planets and satellites: rings – Proto-planetary disks

1. Introduction

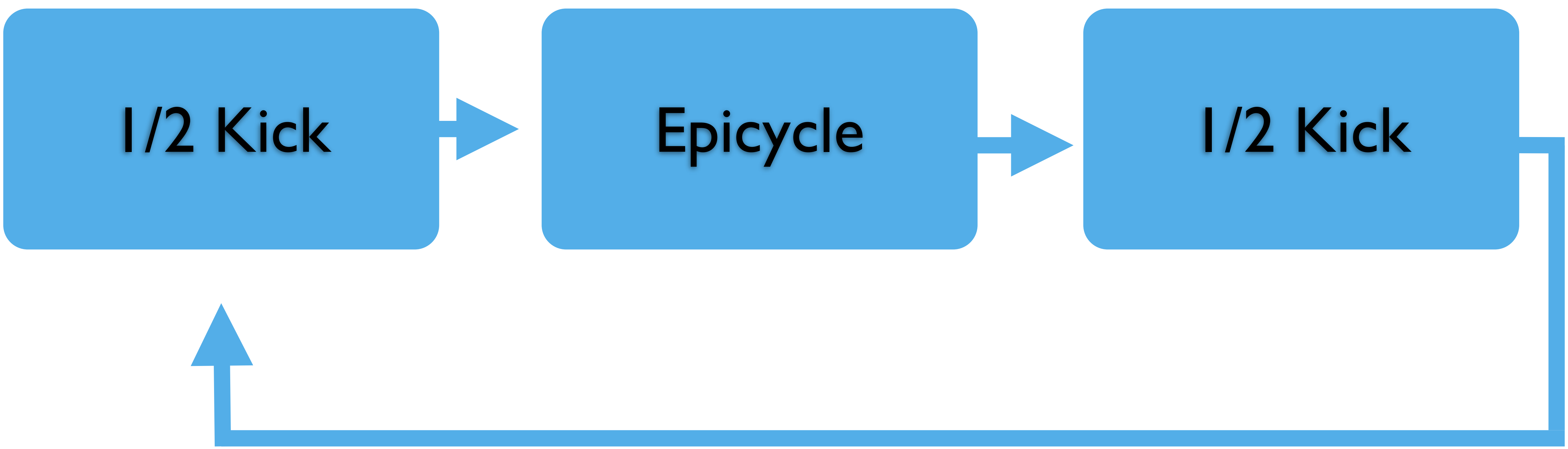
REBOUND is a new open-source collisional N-body code. This code, and precursors of it, have already been used in wide variety of publications (Rein & Papaloizou 2010; Crida et al. 2010; Rein et al. 2010, Rein & Liu in preparation; Rein & Latter in preparation). We believe that REBOUND can be of great use for many different problems and have a wide reach in astrophysics and other disciplines. To our knowledge, there is currently no publicly available code for collisional dynamics capable of solving the problems described in this paper. This is why we decided to make it freely available under the open-source license GPLv3[1].

Collisional N-body simulations are extensively used in astrophysics. A classical application is a planetary ring (see e.g. Wisdom & Tremaine 1988; Salo 1991; Richardson 1994; Lewis & Stewart 2009; Rein & Papaloizou 2010; Michikoshi & Kokubo 2011, and references therein) which have often a collision time-scale that is much shorter than or at least comparable to an orbital time-scale. Self-gravity plays an important role, especially in the dense parts of Saturn's rings (Schmidt et al. 2009). These simulations are usually done in the shearing sheet approximation (Hill 1878).

Collisions are also important during planetesimal formation (Johansen et al. 2007; Rein et al. 2010, Johansen et al. in preparation). Collisions provide the dissipative mechanism to form a planetesimal out of a gravitationally bound swarm of boulders.

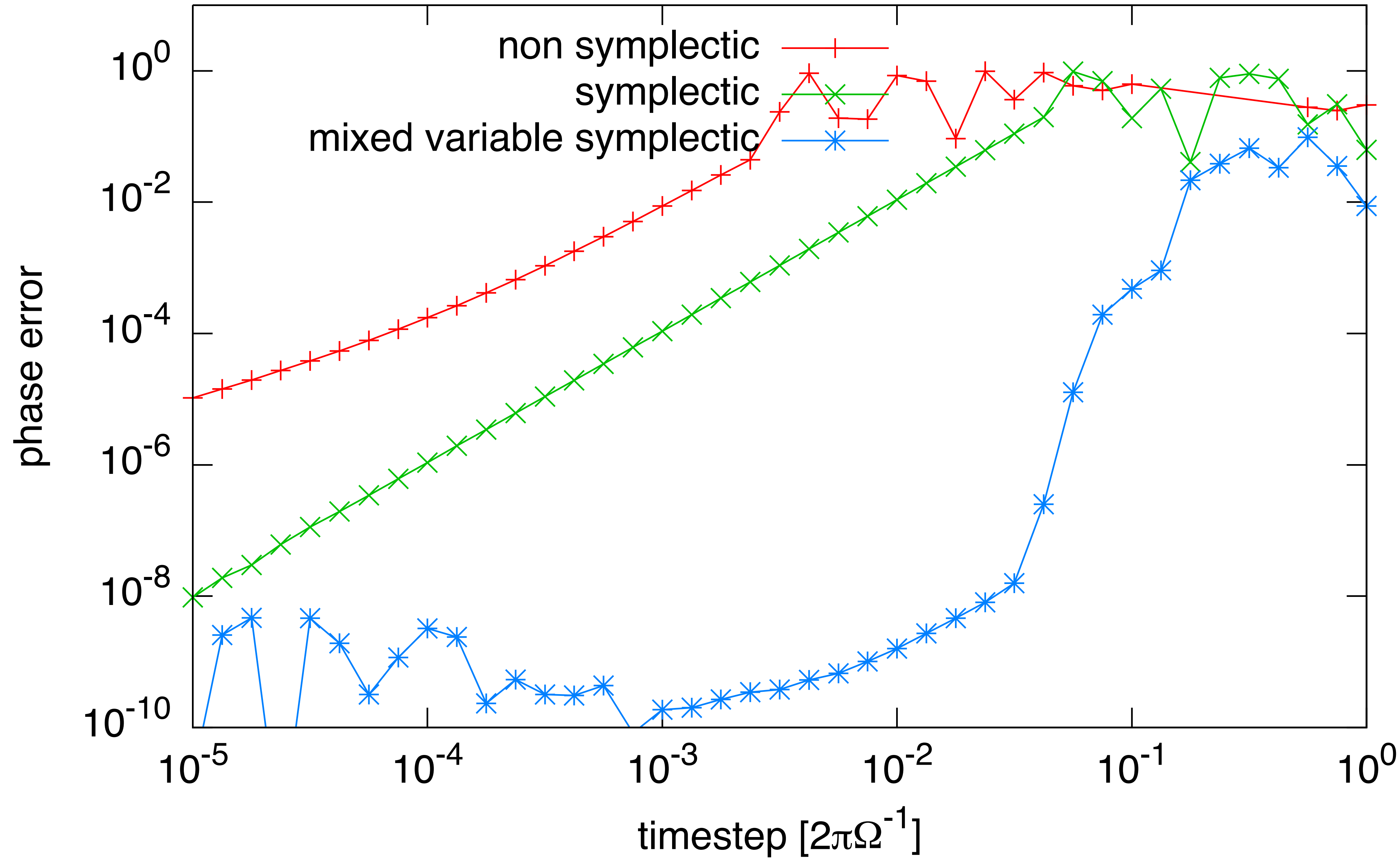
Symplectic Epicycle Integrator

$$H = \underbrace{\frac{1}{2}p^2 + \Omega(p \times r)e_z + \frac{1}{2}\Omega^2\left[r^2 - 3(r \cdot e_x)^2\right]}_{\text{Epicycle}} + \underbrace{\Phi(r)}_{\text{Kick}}$$



Rein & Tremaine 2011

Mixed variable symplectic (MVS) integrator



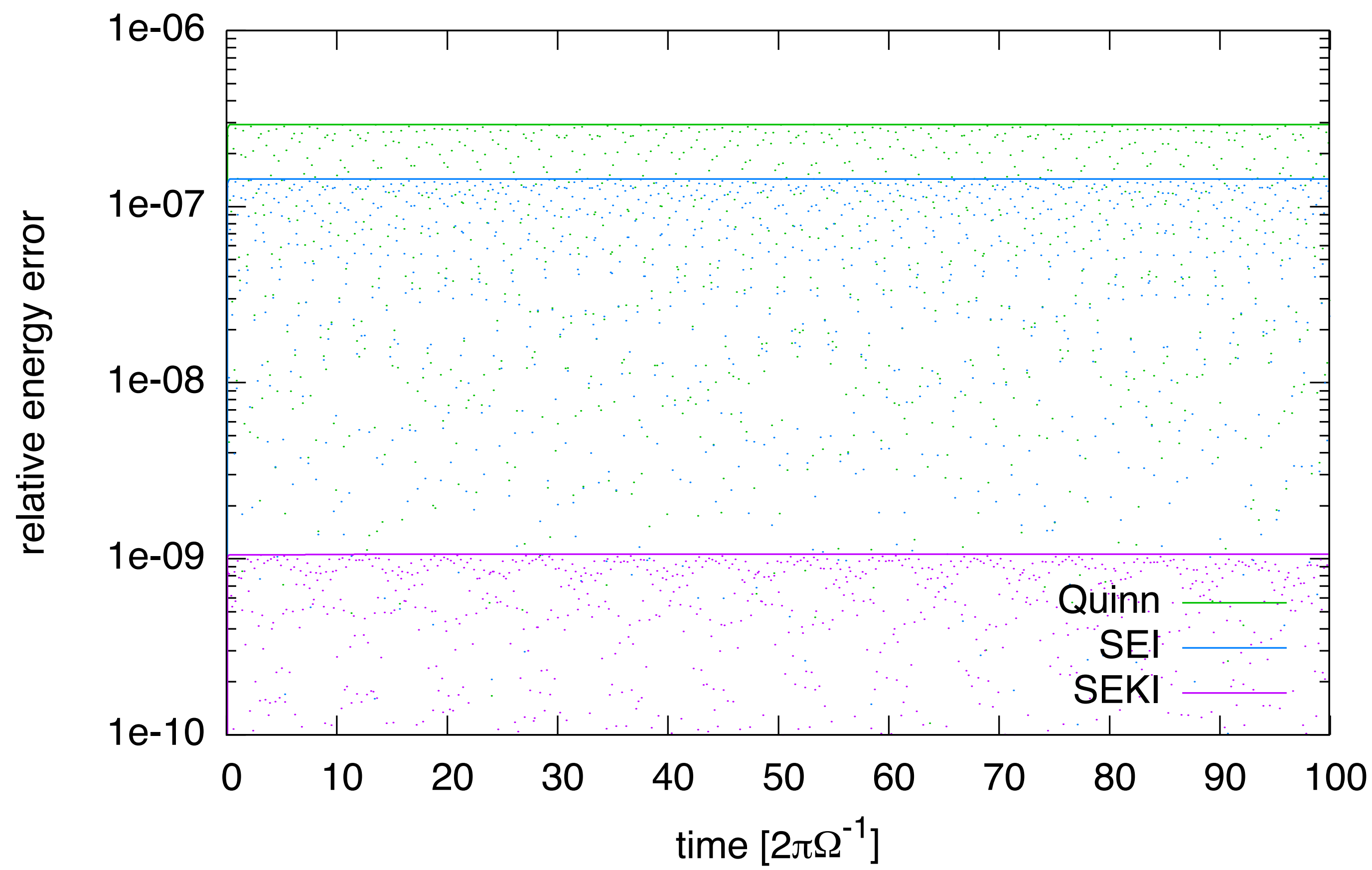
Rein & Tremaine 2011

Symplectic Epicycle Integrator: Rotation

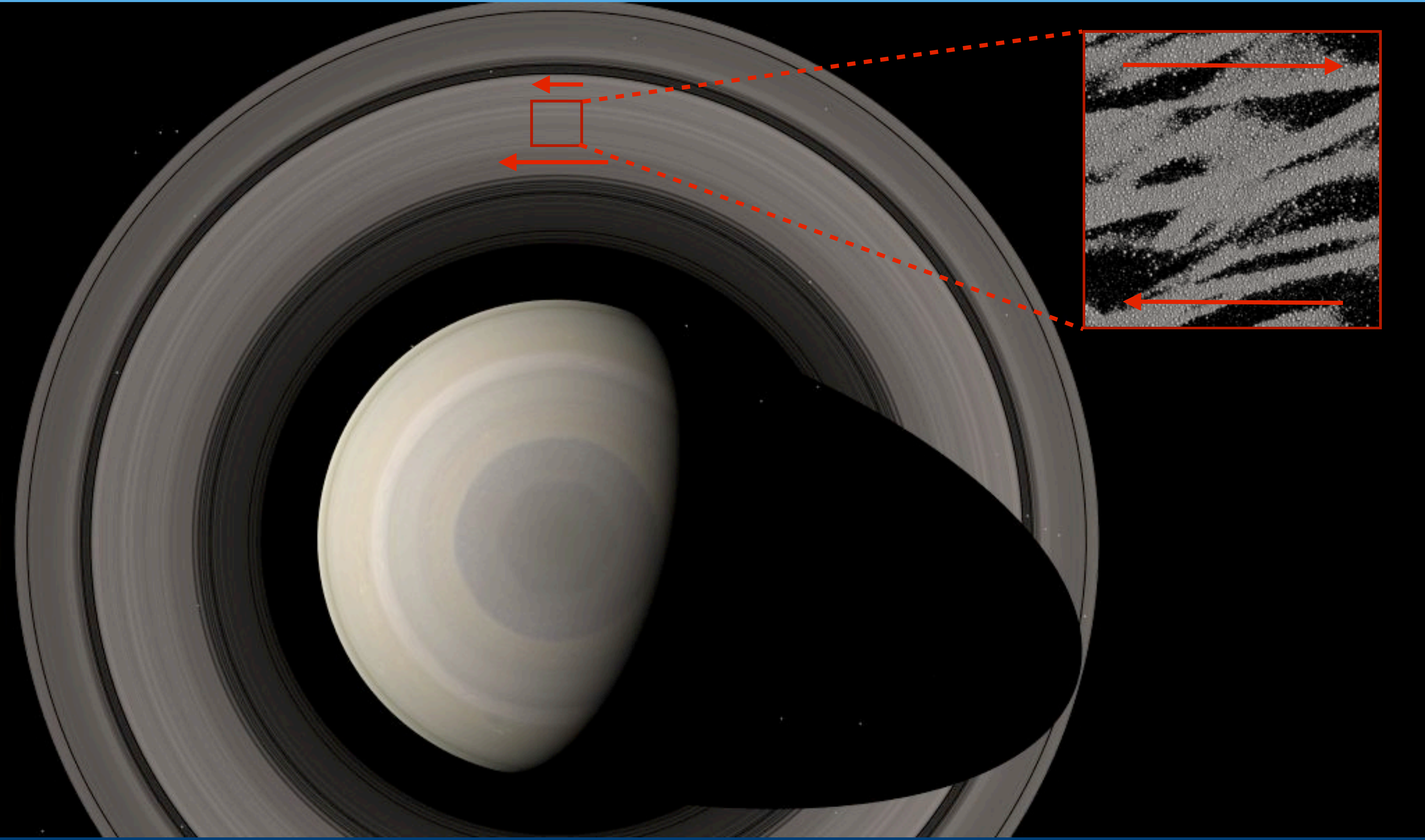
- Solving for the orbital motion involves a rotation.
- Formally $\det(D) = 1$, but due to floating point precision $\det(D) \sim 1$ only.
- Trick: Use three shear operators instead of one rotation.

$$\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\tan\frac{1}{2}\phi & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \sin\phi \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\tan\frac{1}{2}\phi & 1 \end{pmatrix}$$

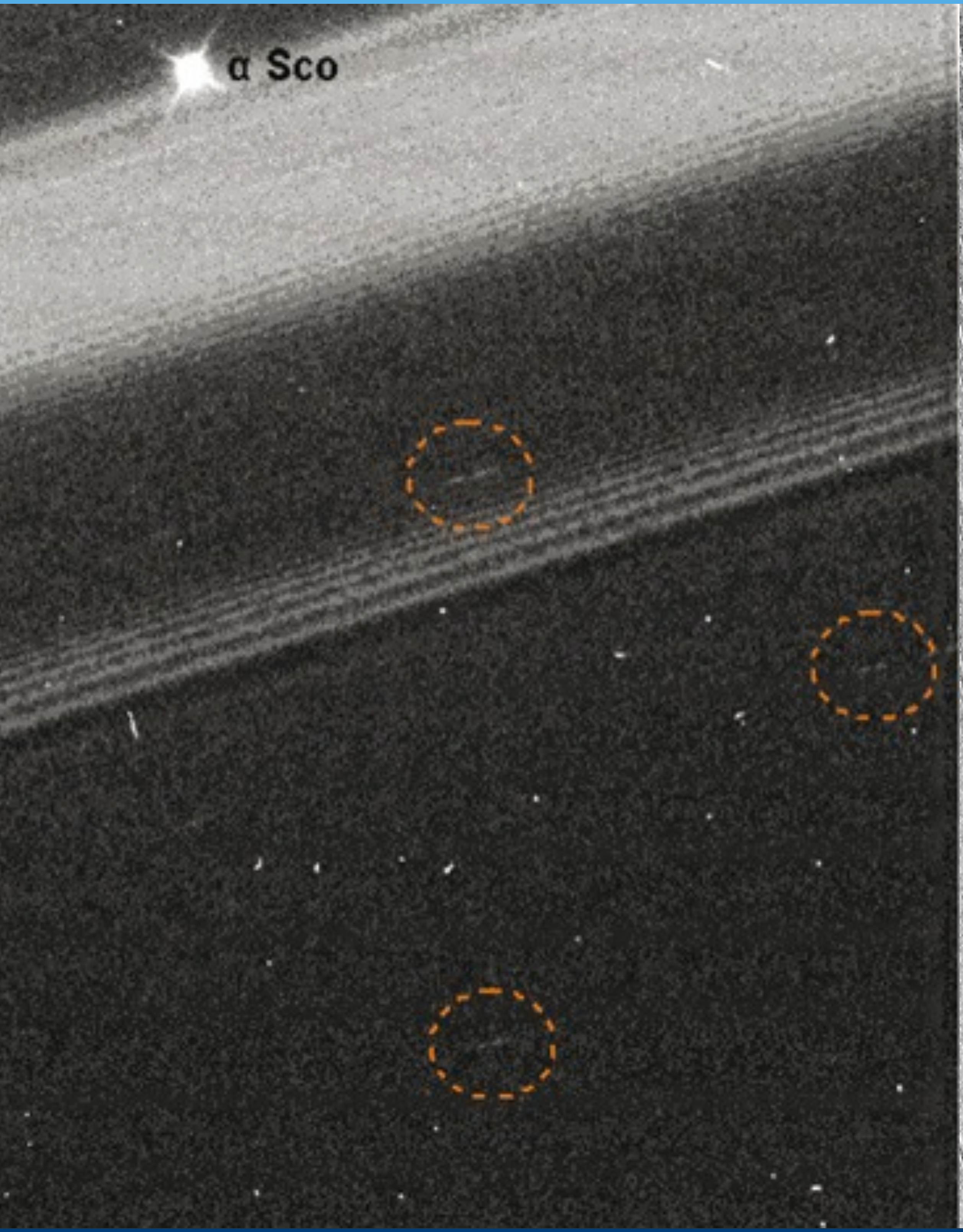
- $\det(D) = 1$ exactly for each shear operator, even in floating point precision.
- No long term trend linear trend anymore!

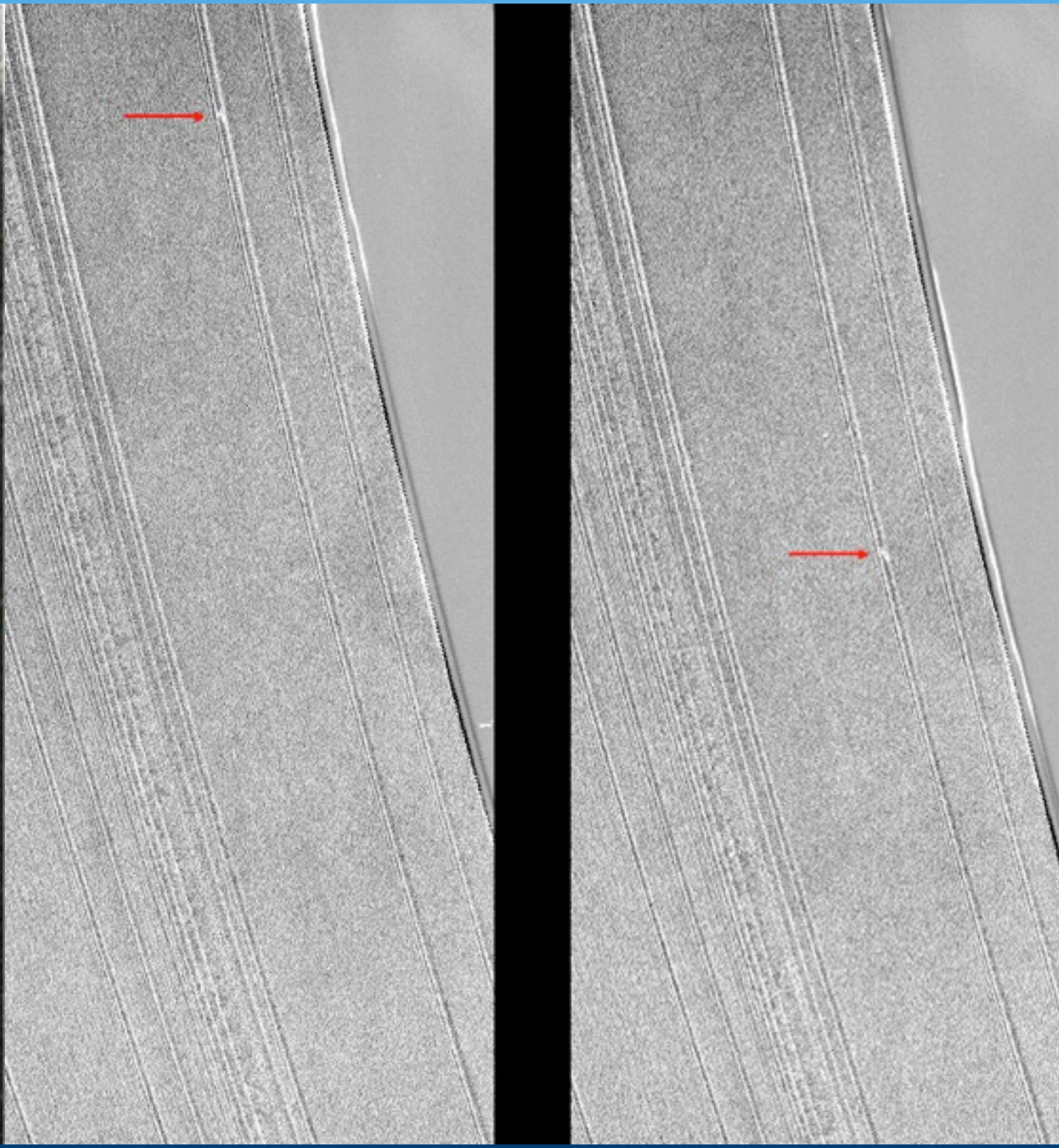


Saturn is a smaller version of the Solar System



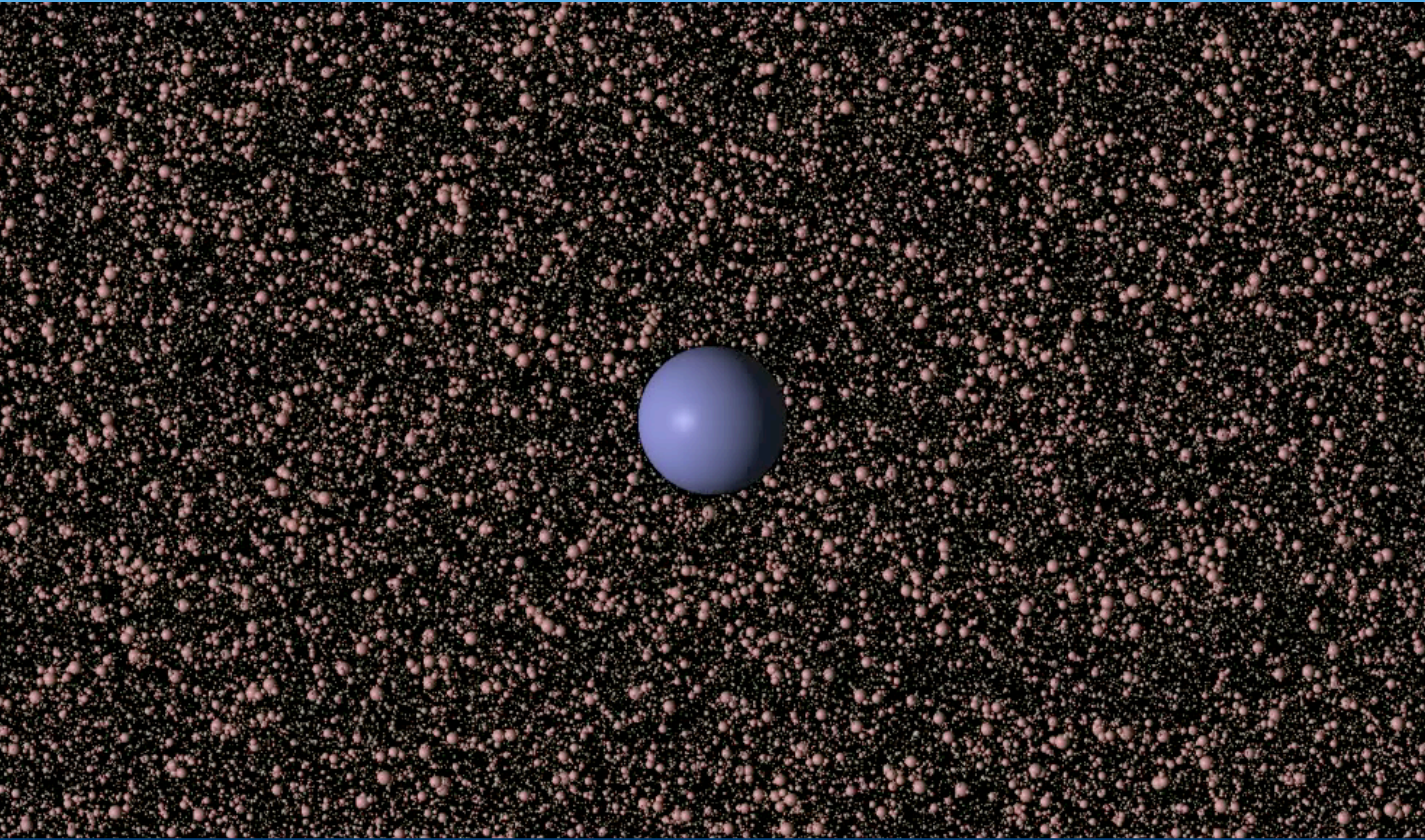
Propeller structures in A-ring





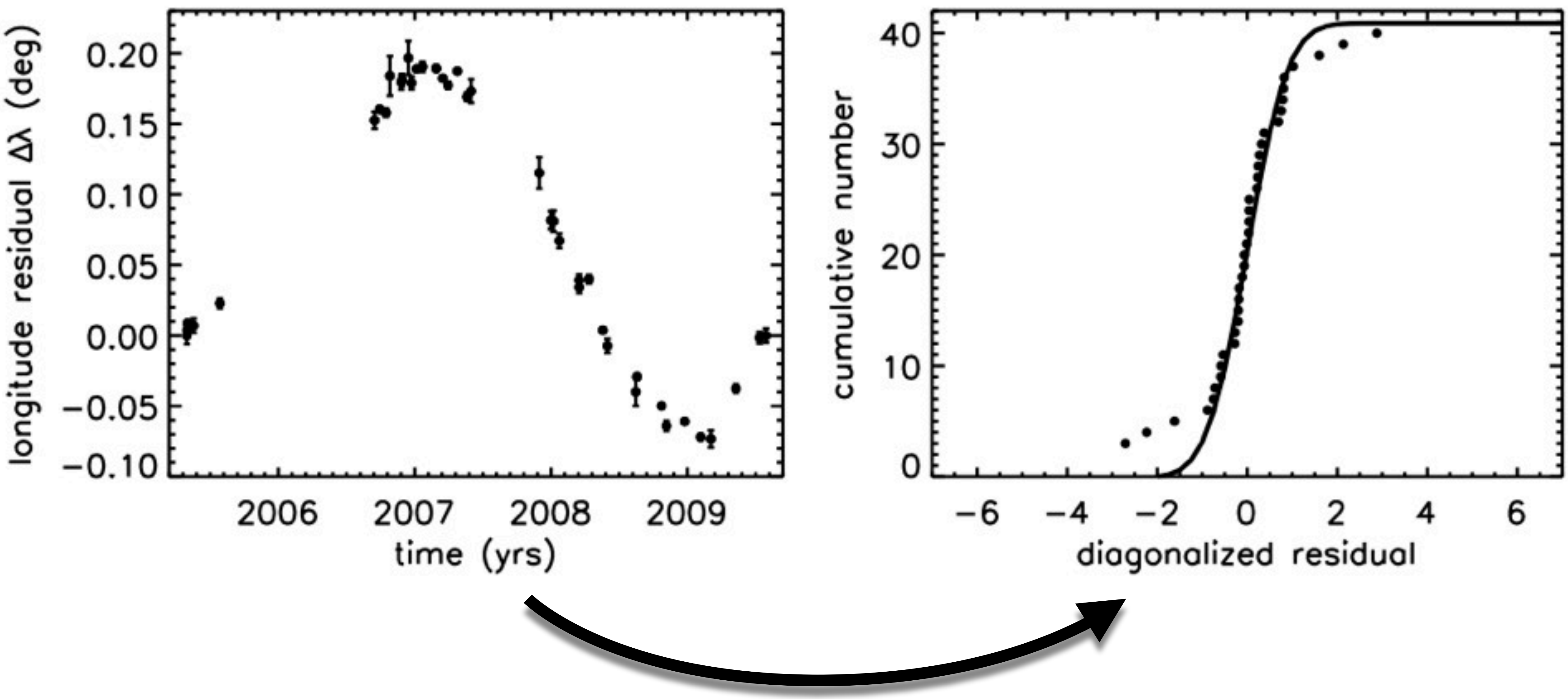
Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006, NASA/JPL-Caltech/Space Science Institute

Stochastic Migration



REBOUND code, Rein & Papaloizou 2010, Crida et al 2010, Pan, Rein, Chiang & Evans 2012

Motion is consistent with a random walk



Diagonalization